OPTIMAL DESIGN OF PREFABRICATED STRUCTURE OF CYLINDRICAL STELL CEILED WITH SEMISPHERICAL DOME

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Abstract A calculation is given for a prefabricated cylindrical tank of piecewise constant thickness, covered by a hemispherical dome, with circular hinges along the guides. Thickness values that satisfy the strength and rigidity conditions are found.

Sh. E. Mikeladze's theory is used to construct a discontinuous solution to the resolving differential equation of the problem.

Key words: spherical shell, cylindrical shell, differential equation, successive approximation, discontinuous solution

Introduction

Technological advances have always necessitated the creation of structures with high strength and rigidity. One such

structure is thin-walled shells. Given their geometric diversity and the impact of loads, where displacements and strains are proportional to the shell thickness, they require the use of nonlinear relationships between strains and displacements, the calculation of which is based on various assumptions and simplifications.

The effectiveness of using thin-walled shells lies in the creation of new computational models and the refinement of calculations.

The development of mathematical and programming methods has enabled us to develop more efficient algorithms.

The problem of calculating a spherical shell has long attracted the attention of scientists both in our country and abroad. Most aspects of this problem have been well-studied and are known in the literature.

However, some issues, crucial for practical applications, remain incompletely understood. For example, when designing spherical domes, interconnected structures are used. In this case, the designer is faced with the task of calculating a shell consisting of ring-shaped elements connected in a specific manner. A similar problem also arises when cracks of sufficient size appear in structures and the distribution of forces must be studied. Calculating such a structure using constraints is complex and, in most cases, practically impossible. The problem is significantly simplified by using the general theory of constructing discontinuous solutions of ordinary differential equations, developed by Sh. Mikeladze.

Main Part

This article examines a cylindrical tank of constant thickness, covered by a hemispherical dome with joints around its circumference. The dome and cylindrical tank are also connected by a circular joint. Regarding the cylindrical shell, it should be understood that, in addition to the liquid, it is subject to transverse and normal forces transmitted from the dome (Fig. 1).

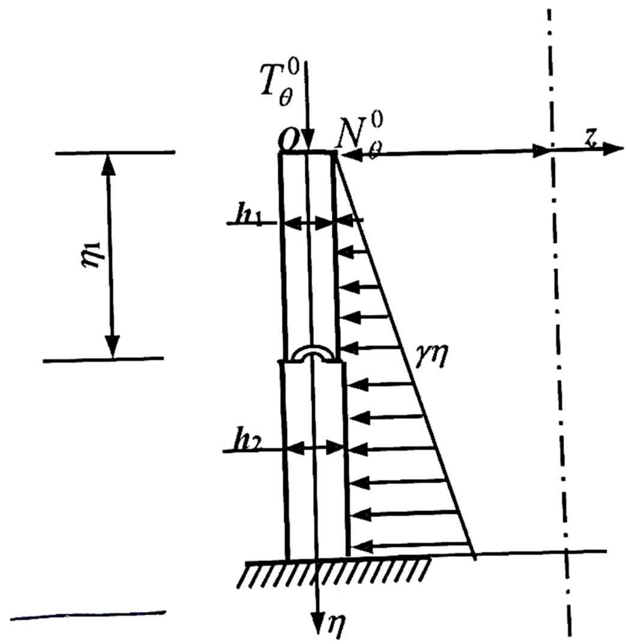

Fig. 1

Optimal design involves selecting the unknown shell thickness in such a way as to satisfy the strength and stiffness conditions imposed on the structure.

$$\frac{d^4W}{d\eta^4}+\frac{1}{h^2}mW=\frac{1}{h^3}B \quad \left(\eta=\frac{x}{1}\right), \tag{1}$$

Where

$$m=\frac{12(1-v^2)1^4}{a^2}, \quad B=\frac{12(1-v^2)1^4}{E}\left(-\frac{T_\theta^0}{a}v-\gamma\eta 1\right).$$

W – Indicates the curve of the shell a – Radius, ℓ – Height, h – Wall thickness, which is a constant value for different sections. E – The appearance of elasticity, γ – Weight by volume of water, T_θ^0 – Normal force transmitted from the dome., v – Poisson's ratio.

Let's place the origin of coordinates at the point where the dome and the reservoir meet.

Let's assume that the shell thickness at this point changes in steps. $h=h_1$ when $0\le\eta\le\eta_1$ and $h=h_2$ when $\eta_1\le\eta\le 1$.

The boundary conditions will be as follows :

$$W(0)=W_0, \quad W'''=-\frac{12(1-v^2)1^3}{Eh_1^3}N_\theta^0, \tag{2}$$

Where N_ϖ^0 This is the shear force transmitted from the dome to the reservoir.

In addition to the five boundary conditions mentioned above, we have two more conditions (strength and stiffness). Namely, bending. η = 0 The point size must be equal to the given value. [W(0)= W_0], And the fastening section And the fastening section (η = 1) Plastic deformation must occur in the peripheral fibers, which indicates the presence of a certain condition. Wrelative to □. The latter is obtained by satisfying the plasticity condition for the specified fibers [1]:

$$X_x^2 + Y_y^2 - X_x Y_y = \sigma_s^2,$$

Where X_x and Y_y The fundamental normal stresses are , and denotes the yield strength of the material.

If we represent stresses under plastic conditions through deformations, we obtain

$$W''(1) = \frac{2(1-v^2)1^2\sigma_s}{h_2 E\sqrt{1-v+v^2}}.$$

This boundary condition allows us to determine four unknown parameters:W(0), W′, $W''(0)$, W''' (0), wo unknown thicknesses: h_1 , h_2 and $W'(\eta)$ -s A_1 jump $\eta = \eta_1$ In terms of relatively $W(\eta)$ and $W''(\eta)$ -ს, They are continuous due to the continuity of the bending moment and bending action.. $W'''(\eta)$ -ს ump ვი $\eta = \eta_1$ In a cross-section, the transverse force is determined from the continuity condition.

$$Q(\eta_{1+}) = Q(\eta_{1-}), \tag{3}$$

Where $Q(\eta_{1+})$ and $Q(\eta_{1-})$ They emphasize the importance of shear force in forming bonds. $\eta = \eta_1$ At the intersection on the right and left. Considering that

$$Q(\eta) = -\frac{Eh^3 W'''(\eta)}{12(1-v^2)},$$

Condition (3) will take the form

$$h_1^3 W'''(\eta_{1-}) = h_2^3 W'''(\eta_{1+}),$$

What is it determined from $W'''(\eta_1)$-That jump

$$A_3 = W'''(\eta_{1-})\left[\left(\frac{h_1}{h_2}\right)^{-3} - 1\right].$$

To integrate equation (1), we rewrite it as follows:

$$\frac{d^4 W}{d^4} = \frac{2}{h^3}B - \frac{1}{h^2}mW$$

Let's search for a solution using the method of successive approximations, where the zeroth approximation W is assumed to be given in the form of a table. We express each new approximation in terms of normal fundamental functions, as provided by the theory of Academician Sh. Mikeladze [2]:

$$W(\eta) = \sum_{k=0}^{3} W^{(k)}(0) Y_{k+1}(\eta) + A_1 Y_2(\eta - \eta_1) + A_3 Y_4(\eta - \eta_1) + \widetilde{W}(\eta), \tag{4}$$

Where $Y_1 = 1$, $Y_2 = \eta$, $Y_3 = 0{,}5\eta^2$, $Y_4 = \frac{1}{6}\eta^3$ normal fundamental functions are,$\widetilde{W}(\eta)$ denotes the partial integral of the inhomogeneous equation, the latter is calculated using the formula [2]

$$\widetilde{W}(\eta) = \int_0^{\eta} Y_4(\eta - t) q(t)dt = \int_0^{\eta} \frac{(\eta - t)^3}{6}\left[\frac{1}{h^3}B - \frac{1}{h^2}mW(t)\right]dt = \frac{1}{6}\left[\frac{1}{h^3}\frac{\eta^4}{4}B + \right.$$

$$\left. + \left(\frac{1}{h_2^3} - \frac{1}{h_1^3}\right)\frac{(\eta_1 - \eta)^4}{4}B - \frac{1}{h_1^2}m\int_0^{\eta}(\eta - t)^3 W(t)dt - m\left(\frac{1}{h_2^2} - \frac{1}{h_1^2}\right)\int_{\eta_1}^{\eta}(\eta - t)^3 W(t)dt\right]. \tag{5}$$

Respectively

$$\widetilde{W}(\eta)=\frac{1}{6}\frac{1}{h_1^3}\eta^3 B+\frac{1}{6}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)(\eta_1-\eta)^3 B+\frac{1}{24}\frac{1}{h_1^3}\eta^4 B'+$$

$$+\frac{1}{24}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)(\eta_1-\eta)^4 B'+\frac{1}{2}\frac{1}{h_1^2}m\int_0^{\eta}(\eta-t)^2W(t)dt-m\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\int_{\eta_1}^{\eta}(\eta-t)^2W(t)dt, \quad (6)$$

$$\widetilde{W}''(\eta)=\frac{1}{3}\frac{1}{h_1^3}\eta^3 B'+\frac{1}{2}\frac{1}{h_1^3}\eta^2 B+\frac{1}{3}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)(\eta_1-\eta)^3 B'+\frac{B}{2}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)(\eta_1-\eta)^2-$$

$$-\left[\frac{1}{h_1^2}m\int_0^{\eta}(\eta-t)W(t)dt+\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)m\int_{\eta_1}^{\eta}(\eta-t)W(t)dt\right]. \quad (7)$$

Based on the boundary conditions for the unknown parameters (2), we obtain the following system of algebraic equations:

$$W_0+W'(0)+\frac{1}{6}W'''(0)+A_1(1-\eta_1)+\frac{1}{6}A_3(1-\eta_1)^3+\widetilde{W}(1)=0, \quad (8)$$

$$W'(0)+0{,}5W'''(0)+A_1+0{,}5A_3(1-\eta_1)^3+\widetilde{W}(1)=0, \quad (9)$$

$$\eta_1 W'''(0)+\widetilde{W}(\eta_1)=0, \quad (10)$$

$$W'''(0)+A_3(1-\eta_1)+\widetilde{W}''(1)=W''(1). \quad (11)$$

Where $\widetilde{W}(1)$, $\widetilde{W}'(1)$, $\widetilde{W}''(\eta_1)$ and $\widetilde{W}''(1)$ Calculated using formulas (5)-(7), as for $W'''(\eta_1)$-ს which A_3 included in the expression, it is calculated based on (4) using the formula

$$W'''(\eta_{1-})=W'''(0)+\widetilde{W}'''(\eta_1),$$

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$$\widetilde{W}'''(\eta)-1{,}5B\frac{1}{h_1^3}\eta^2+B\frac{1}{h_1^3}\eta+1{,}5B'\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)(\eta_1-\eta)-$$

$$-\left[\frac{1}{h_1^2}m\int_0^{\eta}W(t)dt+m\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\int_{\eta_1}^{\eta}W(t)dt\right].$$

System of equations (8)-(11) allows us to determine $W'(0)$, A_1 and unknown h_1, h_2 thicknesses.

As a concrete example, let's consider a shell for which

$$W_0=-0{,}09044,\quad m=608{,}88,\quad B=-0{,}925987-2{,}212358\eta,\quad \nu=0{,}3,\quad a=353{,}55,$$

$$1^2=8a,\quad \nu T_\theta^0=7{,}869928,\quad N_\theta^0=16.414609,\quad W''(1)=-1{,}075755\frac{1}{h_2},\quad W'''(0)=12{,}839412\frac{1}{h_1^3}.$$

Let's get the zero approximation of deflection according to the table (Table 1).

Table 1

η	0,000	0,100	0,200	0,300	0,400	0,500	0,600	0,700	0,800	0,900	1,000
W/W_0	1,000	0,706	0,527	0,518	0,680	0,948	0,746	0,565	0,390	0,210	0,000

We calculate the integrals included in the expressions for the coefficients of the system (8)-(11) using Simpson's formula, and for the zero approximation we have:

$$\int_0^{0,5}(0,5-t)\frac{W(t)}{W_0}dt=0,083200 \qquad \int_0^{1,0}(1-t)^3\frac{W(t)}{W_0}dt=0,0811764$$

$$\int_0^{1,0}(1-t)^3\frac{W(t)}{W_0}dt=0,174667 \qquad \int_0^{1,0}(1-t)^2\frac{W(t)}{W_0}dt=0,029425$$

$$\int_0^{1,0}(1-t)\frac{W(t)}{W_0}dt=0,22987500 \qquad \int_{0,5}^{1,0}(1-t)\frac{W(t)}{W_0}dt=0,078686 \qquad (12)$$

$$\int_0^{1,0}(1-t)\frac{W(t)}{W_0}dt=0,335387 \qquad \int_{0,0}^{0,5}\frac{W(t)}{W_0}dt=0,34337$$

Considering them

$$\widetilde{W}(1)=-0{,}122591\frac{1}{h_1^3}-0{,}008173\frac{1}{h_2^3}+1{,}716095\frac{1}{h_1^2}+0{,}123923\frac{1}{h_2^2},$$

$$\widetilde{W}'(1)=-0{,}674860\frac{1}{h_1^3}-0{,}059621\frac{1}{h_2^3}+6{,}334898\frac{1}{h_1^2}+0{,}929933\frac{1}{h_2^2},$$

$$\widetilde{W}''(1)=-2{,}006514\frac{1}{h_1^3}-0{,}300116\frac{1}{h_2^3}+16{,}225282\frac{1}{h_1^2}+4{,}973474\frac{1}{h_2^2},$$

$$\widetilde{W}''(0{,}5)=-0{,}346202\frac{1}{h_1^3}+5{,}258839\frac{1}{h_1^2},$$

$$\widetilde{W}'''(0{,}5)=-1{,}845773\frac{1}{h_1^3}+22{,}501460\frac{1}{h_1^2},$$

$$A_3=\left[\left(\frac{h_1}{h_2}\right)^3-1\right]\left(-10,993638\frac{1}{h_1^3}+22,501465\frac{1}{h_1^2}\right).$$

Accordingly h_1, h_2, $W'(0)$ and A_1 -we will have the following system for it:

$$-0{,}5(12{,}839412)\frac{1}{h_1^3}+\left(00,345292\frac{1}{h_1^3}+5,258839\frac{1}{h_1^2}\right)=0,$$

$$-12,839412\frac{1}{h_1^3}+0,5\left[\left(\frac{h_1}{h_2}\right)^3-1\right]\left(-10,993639\frac{1}{h_1^3}+22,501465\frac{1}{h_1^2}\right)-2,006514\frac{1}{h_1^3}-$$

$$-0,300116\frac{1}{h_2^3}+16,225282\frac{1}{h_1^2}+4,973474\frac{1}{h_2^2}+1,075865\frac{1}{h_1}=0$$

$$W'(0)+6,419706\frac{1}{h_1^3}+A_1+0,5\left[\left(\frac{h_1}{h_2}\right)^3-1\right]\left(-10,993639\frac{1}{h_1^3}+22,501465\frac{1}{h_1^2}\right)-$$

$$-0,674860\frac{1}{h_1^3}+0,059621-\frac{1}{h_1^3}+6,334898\frac{1}{h_1^2}+0,929933\frac{1}{h_2^2}=0$$

$$-0,09044+2,1399092\frac{1}{h_1^3}+W'(0)+0,5A_1+\frac{1}{6}\left[\left(\frac{h_1}{h_2}\right)^3-1\right]\left(-10,993639\frac{1}{h_1^3}+\right.$$

$$\left.+22,501465\frac{1}{h_1^2}\right)-0,122591\frac{1}{h_1^3}-0,008173\frac{1}{h_2^3}+1,716095\frac{1}{h_1^2}+0,123923\frac{1}{h_2^2}=0$$

Solving the system gives us: h_1=1,28657, h_2= 4,709632, $W'(0)$ = 0,212741, $A1$ = –0,069213.

As a first approximation, deflections are calculated using the following formula: $W(\eta)=W_0+\eta W'(0)+\frac{1}{6}\eta^3W'''(0)+A_1(\eta-\eta_1)^3+\widetilde{W}(\eta)$,

where

$$\widetilde{W}(\eta)=\frac{1}{24}\frac{1}{h_1^3}B\eta^4+\frac{1}{24}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)B(\eta_1-\eta)^4+\frac{1}{6}m\frac{1}{h_1^2}\int_0^{\eta}(\eta-t)^3\frac{W(t)}{W_0}dt+$$
$$+\frac{1}{6}m\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\int_{\eta_1}^{\eta}(\eta-t)^3\frac{W(t)}{W_0}dt\,.$$

The calculation results are presented in tables (Tables 2, 3).

Taking into account the first approximation of the deflections, we calculate integrals (12), as well as the unknown thicknesses and parameters from the system of algebraic equations. Using the obtained parameters and thicknesses, we find the next approximation for the deflections, and so on, until the thickness values match each other with the previously required accuracy. This time, it was necessary to perform six approximations, which ensured that the thicknesses were determined with an accuracy of three significant digits. The final results are presented in the table (Table 4).

Table 2

η	$\frac{1}{24}\frac{1}{h_3^1}B\eta^4$	$\frac{B(\eta_1-\eta)}{24}\left(\frac{1}{h_2^3}-\frac{1}{h_1^3}\right)$	$\frac{m}{6h_1^2}\int_0^{\eta}(\eta-t)^3\frac{W(t)}{W_0}dt$	$\frac{m}{6}\left(\frac{1}{h_2^2}-\frac{1}{h_1^2}\right)\int_0^{\eta}(\eta-t)^3\frac{W(t)}{W_0}dt$	$\widetilde{W}(\eta)$
0,00	0,00000000		0,00000000		0,00000000
0,10	–0,00000224		0,00021001		0,00020777
0,20	–0,00004284		0,00229593		0,00225309
0,30	–0,00025193		0,01074219		0,01049026
0,40	–0,00090703		0,03197476		0,03106773
0,50	–0,00248496	0,00000000	0,07468840	0,00000000	0,07220344
0,60	–0,00571378	0,00000192	0,14997600	–0,00186071	0,14244038
0,70	–0,01162476	0,000030667	0,27275439	–0,00207441	0,25908588
0,80	–0,02160427	0,00015525	0,3722766	–0,00993254	0,4036582
0,90	–0,03744580	0,00049065	0,73304273	–0,02980741	0,66628018
1,00	–0,06140180	0,001197880	1,11160329	–0,06927820	0,98212117

Table 3

η	$\eta W'(0)$	$\frac{1}{6}\eta^2 W'''(0)$	$A_1(\eta-\eta_1)$	$\frac{1}{6}A_3(\eta-\eta_1)^3$	$W(\eta)$	$W(\eta)/W_0$
0,00	0,00000000	–0,00000000			–0,09044000	1,0000
0,10	0,02127414	–0,00100481			–0,06996289	0,7736
0,20	0,04254828	–0,00803851			–0,05367712	0,5935
0,30	0,06382242	–0,02712996			–0,04325727	0,4783
0,40	0,08509656	–0,06430805			–0,03858375	0,4266
0,50	0,10637070	–0,12560166	0,00000000	0,00000000	–0,03746749	0,4143
0,60	0,12764484	–0,21703967	0,00692134	–0,00109360	–0,03156669	0,3491
0,70	0,14891898	–0,34465096	0,01384268	–0,00874882	–0,02199221	0,2432
0,80	0,17019312	–0,51446441	0,03981632	–0,03981632	–0,0102890	0,1138
0,90	0,19146726	–0,73250889	0,02768537	–0,06999057	–0,00750662	0,0830
1,00	0,212274140	–1,00232901	0,03460671	–0,13670031	–0,00000000	0,0000

Table 4

Approach	0	1	2	3	4	5	6
h_1	1,286	1,253	1,244	1,242	1,223	1,221	1,221
h_2	4,709	4,986	4,024	3,905	3,9079	3,906	3,906
A_1	0,069	0,086	0,0905	0,09777	0,1085	0,1087	0,1089
$W'(0)$	0,213	0,2113	0,2119	0,2117	0,2115	0,211	0,211

A reservoir whose thickness changes abruptly from two to three times is calculated in a similar manner. $\eta=\eta_1$ and $\eta=\eta_2$ in points. (Fig. 2) The final results are shown in the table (Table 5).

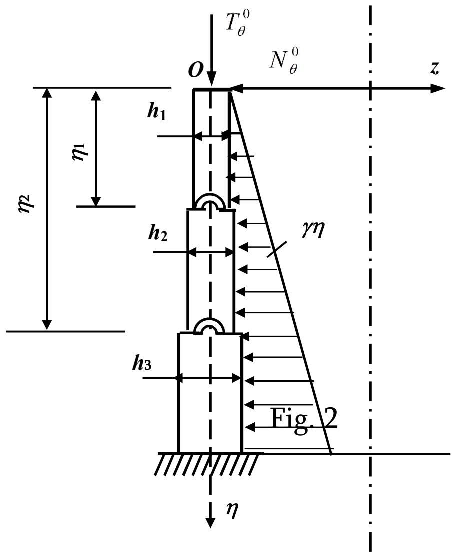

Fig. 2

Table 5

Approach	0	1	2	3	4	5	6
h_1	1,328	1,378	1,382	1,38	1,382	1,382	1.382
h_2	4,057	3,743	3,769	3,767	3,778	3,775	3,775
h_3	5,412	6,222	6,38	6,338	6.447	6,445	6,445
A_1	0,272	0,256	0,252	0,254	0,254	0,254	0,254
A'_1	–0,391	–0,354	–0,344	–0,352	–0,352	–0,352	–0,352

Conclusion

The calculation results showed that six approximations are possible, which ensures thickness determination with an accuracy of up to three significant digits after the decimal point, while for practical purposes, three approximations are required to determine thickness with an accuracy of up to two significant digits after the decimal point.

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