Algorithms for Investigating the Strength, Stability, and Vibrations of Assembled Structures and Their Individual Elements

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Abstract This paper presents a comprehensive study of algorithms for analyzing the strength, stability, and vibration behavior of assembled structural systems and their individual elements. The information required for static strength analysis is formulated and processed using dedicated computational programs, transforming initial data into standardized forms suitable for algorithmic implementation. Numerical examples are presented, and the obtained results are critically analyzed, demonstrating the efficiency and reliability of the proposed approaches.

Keywords: algorithm; structural systems; strength; stability; vibrations.

1. Introduction

The proposed algorithms determine nodal displacements of structures in plane problems under asymmetric loading and in spatially symmetric problems with consideration of geometric nonlinearity. The obtained results have both independent significance and the role of initial input data for subsequent computational procedures. Algorithms for analyzing the strength of individual structural elements may be applied independently or as part of an integrated algorithmic framework addressing complex strength problems.

Algorithms intended for the investigation of stability and vibration behavior of symmetrically loaded structures (spatially symmetric systems) employ the results of nodal displacement determination obtained under symmetric loading. These results are used to define the critical stress–strain state. All presented algorithms may be applied to structures whose element properties can be described using geometric modeling and mesh-based procedures.

For the efficient solution of complex optimization problems, thorough analysis of available information on design projects and possible structural modifications is essential. Particular importance is assigned to incomplete input data and internal effects caused by various factors that may possess deterministic characteristics. However, for real structural systems, obtaining such information is closely associated with the application of efficient numerical methods.

One effective approach is design sensitivity analysis. This method forms the basis of techniques relying on structural optimality and iterative solution procedures. The most general design methodology includes decisions made by the engineer during the design process. This widely accepted approach can be significantly improved when information on structural sensitivity is available.

In such cases, systematic structural analysis and rational design improvement become possible. This information can be obtained using methods based on finite element formulations. The computational efficiency of sensitivity analysis in design depends on the representation of elements (in the present case, beam and bending elements). The availability of interactive graphical methods and modern operating systems enables computer-aided interactive design. Based on these considerations, the relevance of the present study is evident.

2. Main Part

Typically, design variables are represented by cross-sectional characteristics of beam elements, geometric parameters of shell elements, as well as their mechanical and elastic properties. Consequently, the global stiffness matrix K and the generalized load vector F are functions of the design variables, namely:

$$K = K(b), \quad F = F(b)$$

where b denotes the vector of design variables characterizing the dimensions, mechanical properties, and nodal parameters of structural elements. In this study, the case in which boundary conditions do not explicitly depend on design variables is considered. Under this assumption, optimization procedures based on sensitivity analysis may be successfully applied.

The essence of the method is as follows: since the global stiffness matrix and load vector depend on the design variables, both linear and nonlinear forms of the governing equations are also dependent on them. Consequently, the displacement vector z becomes a function of the design variables, i.e.,

$$z = z(b).$$

In optimal structural design problems, either minimization or maximization of an objective function is required, subject to constraints on stresses, displacements, and design variables. The objective function may represent any criterion of optimal design. After solving the equilibrium equations, the dependence of the objective function on the design variables may be explicit or implicit. The purpose of sensitivity analysis is to determine the complete dependence of such functions on the design variables.

By applying differentiation rules for composite functions and matrix calculus, derivatives of the governing equations are obtained. Since the stiffness matrix is non-singular, the derivatives of nodal displacements with respect to design variables can be efficiently computed, enabling the effective application of gradient-based optimization techniques.

The application of this method to specific optimization procedures is not discussed in detail, as it lies beyond the scope of the present paper.

Any computational process performed on a computer is characterized by parameters such as static and dynamic memory requirements. In the present study, comparison is conducted not only for the complete structural analysis process but also for the stage involving determination of the stiffness matrix of bending elements. Exact methods based on numerical integration of systems of first-order differential equations are compared with approximate methods employing elementary functions that provide explicit expressions for stiffness matrix coefficients.

In the first case, in addition to static memory, dynamic memory is required, depending on the number of orthogonalization points along the entire length of the element. In contrast, approximate methods require only static memory. The performed comparisons demonstrate that the ratio of required memory volumes can be expressed as:

$$W\_exact / W\_approx = 1.07m - 0.035$$

where W_exact denotes the total memory required for numerical integration, W_approx is the memory required for the approximate method, and m is the number of orthogonalization points.

It follows from this relation that for elements of medium length, the required memory is practically independent of the applied method. For long elements (m > 40), the required memory for exact methods increases by approximately 2.5 times and reaches about 2.5 kB. Although this volume is not critical for modern computing systems, in complex software packages it may influence overall computational efficiency.

An analysis of computational time required for determining stiffness matrices using exact and approximate methods is performed over a wide range of parameters. The obtained results indicate that approximate methods provide significant time savings—up to two orders of magnitude for long elements—making them particularly advantageous for multi-variant optimal structural design.
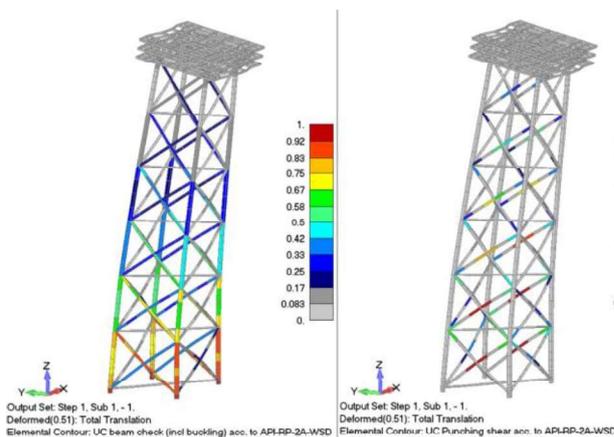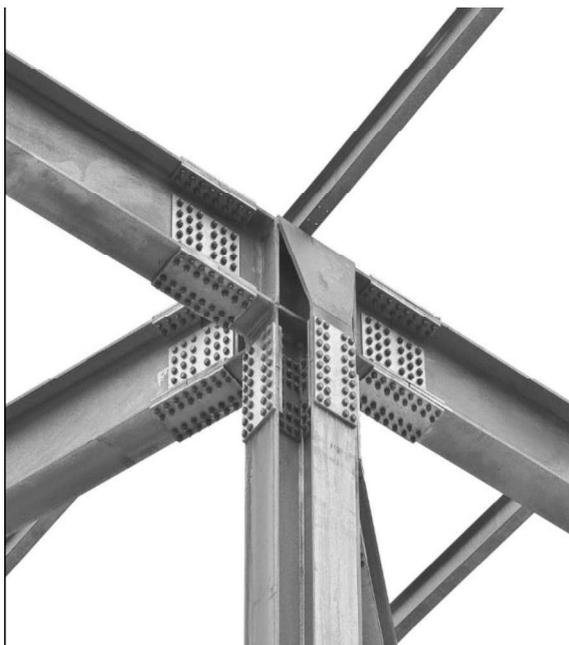
Thus, the use of approximate methods for determining stiffness matrices of bending elements ensures acceptable accuracy while being substantially more

efficient than exact methods in terms of both memory consumption and computational speed. This clearly demonstrates the superiority of the approximate approach.
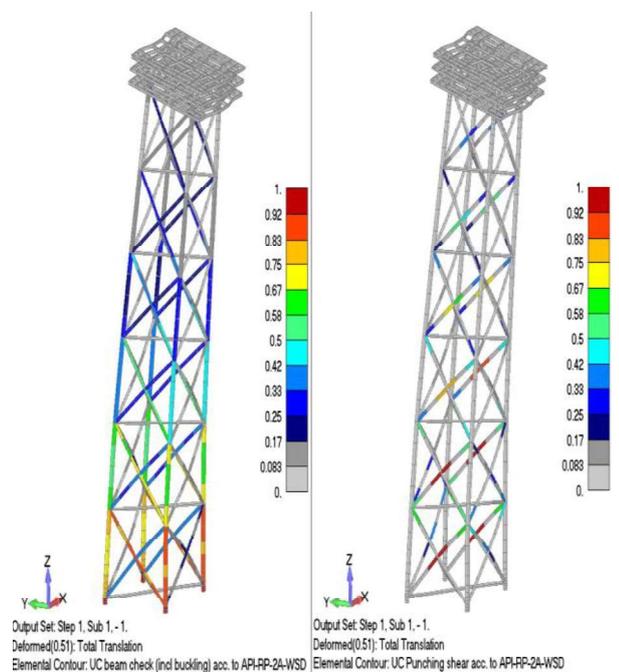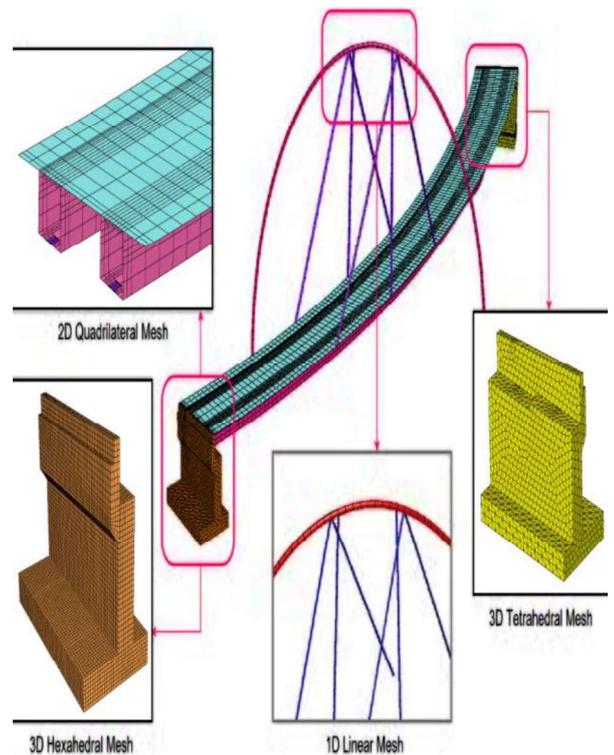
Conclusions

- A new approach to the optimal design of multilayer plates has been developed.
- Programs for processing initial stress–strain state information have been created.
- Information required for static strength analysis of assembled structures has been formulated and standardized for use in the proposed algorithms.
- Examples of structural calculations have been examined and the obtained results have been analyzed.
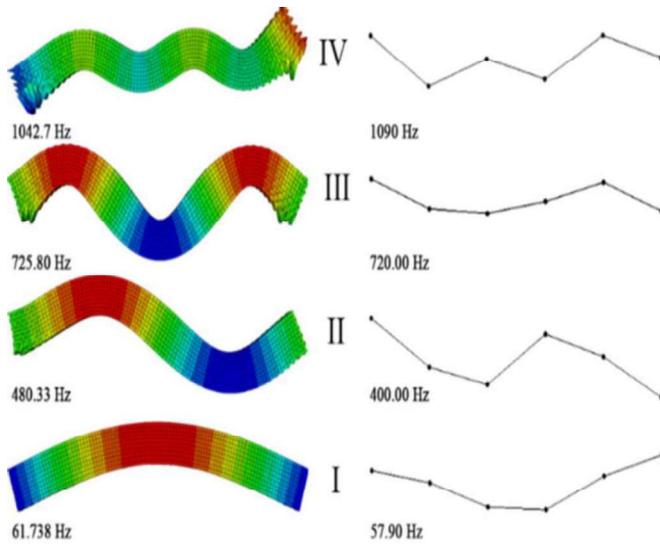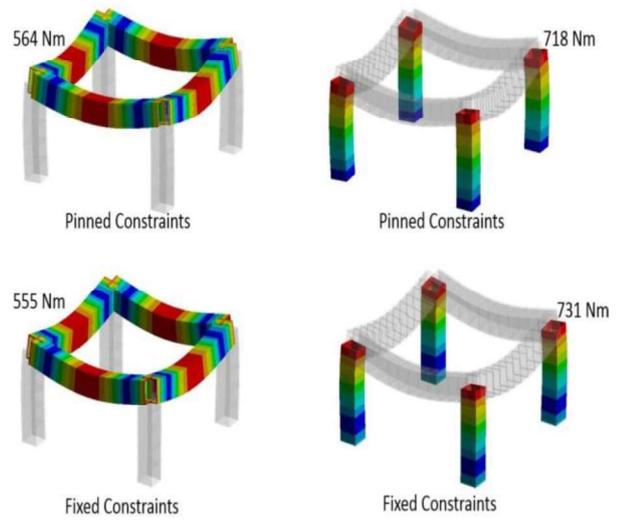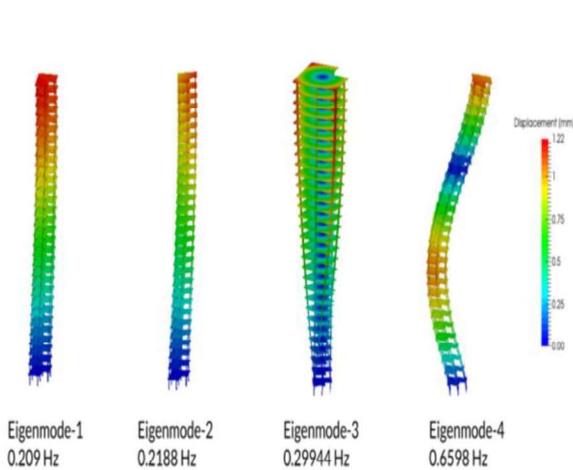
- **Assembled / Frame Structures (Beam–Column Systems**

- **inite Element Method (FEM) – Mesh & Structural Model**

2D Quadrilateral Mesh

3D Tetrahedral Mesh

3D Hexahedral Mesh

1D Linear Mesh

Output Set: Step 1, Sub 1, - 1.
Deformed(0.51): Total Translation
Elemental Contour: UC beam check (incl buckling) acc. to API-RP-2A-WSD

Output Set: Step 1, Sub 1, - 1.
Deformed(0.51): Total Translation
Elemental Contour: UC Punching shear acc. to API-RP-2A-WSD

Output Set: Step 1, Sub 1, - 1.
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Output Set: Step 1, Sub 1, - 1.
Deformed(0.51): Total Translation
Elemental Contour: UC Punching shear acc. to API-RP-2A-WSD

- *Vibration / Modal Analysis*

Eigenmode-1
0.209 Hz

Eigenmode-2
0.2188 Hz

Eigenmode-3
0.29944 Hz

Eigenmode-4
0.6598 Hz

564 Nm

Pinned Constraints

718 Nm

Pinned Constraints

555 Nm

Fixed Constraints

731 Nm

Fixed Constraints

IV
1042.7 Hz   1090 Hz

III
725.80 Hz   720.00 Hz

II
480.33 Hz   400.00 Hz

I
61.738 Hz   57.90 Hz

- *Multilayer Plates / Shell Structures*

Shell Models
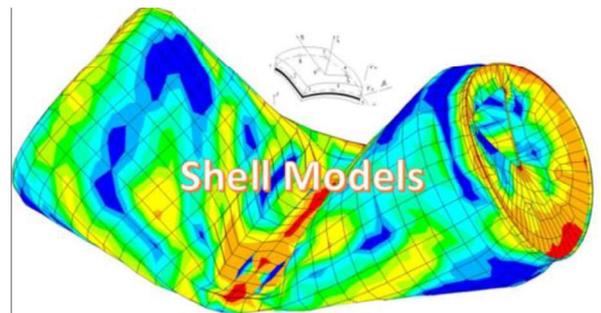
- *uckling / Stability Mode Shape*

Study: "Buckling"
Relative Displacement, magnitude
Buckling Mode 8 - Load Factor: 20.62

Study: "Buckling"
Relative Displacement, magnitude
Buckling Mode 5 - Load Factor: 16.75
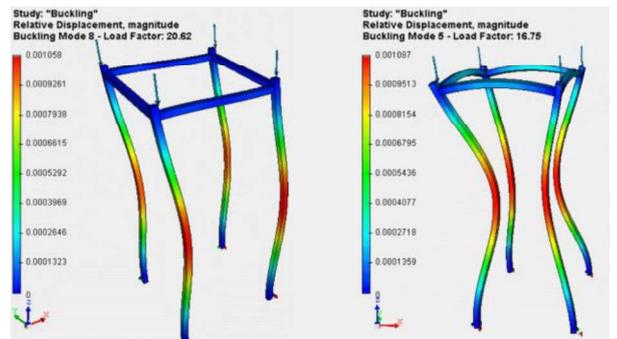
- *Sensitivity Analysis / Optimization Flow*

Local Sensitivity Analysis

Derivative-based Global Sensitivity Analysis

Variance-based Global Sensitivity Analysis

$Model: f(\theta) \rightarrow z$

$Metrics: \frac{\partial z}{\partial \theta_i} (i=1,...,n_\theta)$

$Metrics: \int \left(\frac{\partial z}{\partial \theta_i}\right)^2 \rho(\theta) d\theta$
$(i=1,...,n_\theta)$

$Metrics: Sobol'\ indices$
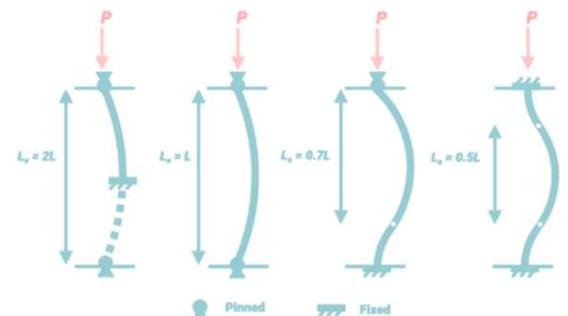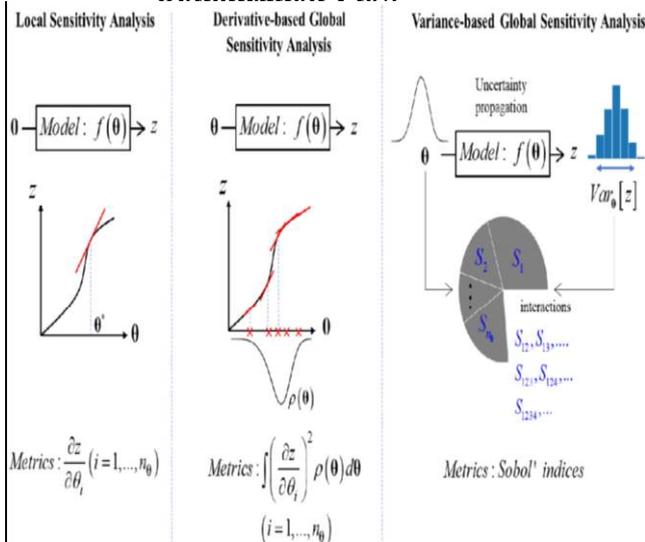
- ***Modern Structural Frame Buildings***

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