.Research on Cracks Caused by Nonlinear Deformation of a Reinforced Concrete Slab

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Abstract The article develops a software package based on a calculation method for cracks caused by nonlinear deformations in reinforced concrete slabs with discontinuous parameters. The system's functionality is based on a mathematical apparatus for modeling cracks in reinforced concrete slabs. The subsystem is presented as a package of problem-oriented procedures for determining the Stress-Strain State (SSS), along with a subsystem of mathematical and software algorithms for SSS determination, which is housed in a library of load modules. The subsystem software consists of a package of applied software modules that perform specific mathematical and logical functions. A new method for calculating the deformation and stability of plates with discontinuous-parameter cracks has been developed. Based on this method, calculation results for specific problems have been obtained. The effectiveness of the method lies in the use of a standard set of approximating functions at each loading stage; these functions represent solutions to linear problems and can be constructed using any known calculation methods.

Keywords: Plate, Crack, Model, Software, Deformability.

Introduction

In the modern construction industry, significant importance should be assigned to works aimed at researching the physical nature of the strength and deformation capacity of concrete and reinforced concrete. Such research reveals the resistance mechanisms of these materials to the deforming and destructive impacts of structural and other factors. Specifically, from the perspective of modern solid-state fracture mechanics, the study of the patterns of formation and development of various types of structural cracks in concrete and reinforced concrete is of great interest. The strength and deformation capacity of concrete and reinforced concrete—specifically the intensity of deformation development under constant loads, long-term strength in aggressive environments, the endurance limit, and similar issues—depend on the structural cracks formed and accumulated within the concrete.

Based on the analysis of the research results, a study of cracks caused by the nonlinear deformations of reinforced concrete slabs has been developed. We have studied the methodology, the deformed state of concrete and reinforced concrete, and the process of crack formation. [1]

The patterns of crack formation and development, as well as the quantitative assessment of their magnitudes based on the degree of reinforcement and the type of aggregate, are presented in works [1; 2; 3].

Experimental studies and their comparisons, conducted on the basis of theoretically obtained results, are presented in work [4].

One of the primary issues determining the deformation and crack resistance of reinforced concrete (RC) slab structures is the consideration of the specific properties of reinforced concrete. As a two-component structural material, reinforced concrete differs from other materials due to several characteristic properties. For instance, properties such as heterogeneity, anisotropy, and crack initiation create certain difficulties in the calculation and design of building structures.

Various deformation models are currently used to describe the deformation and failure of RC slab structures. They can be conventionally divided into the following groups:

The First Group: Models in which the stress-strain state of a two-component composite material is considered at a point separately for concrete and separately for reinforcement. In this case, the physical equation for a section or a characteristic element is formulated based on the combined deformation of the composite, which includes concrete and steel. Furthermore, the elastic or elasto-plastic deformations of the reinforced concrete are accounted for in the relationships obtained according to the calculated state diagrams. The merit of these models lies in their generality and universality.

The Second Group: This group includes so-called microstructural deformation models. The idea behind these models, initially considered for the construction of models for frame elements, involves examining a characteristic area of the RC element between two adjacent cracks, where the averaged deformations of the reinforcement and concrete are determined using a special procedure.

For the development of a general theory for calculating RC structures, it was of great importance to reduce the problem of the stress-strain state of an RC element to the calculation of a bar with variable stiffness and to apply conventional methods of structural mechanics. The integral deformation model is determined by the loading level, regime, and duration, as well as by the strength and deformation characteristics and the cross-section.

The Third Group: This group of physical models includes so-called block models of RC deformation. The concept behind these models represents the structure as being composed of distinct blocks.

Although block models allow for a more detailed description of deformation processes in cracked sections, the solution to the contact problem of reinforcement-to-concrete bonding still relies on empirical relationships for determining bond forces and the shear modulus. Based on the above, this work is undoubtedly of high relevance.

Main Part "The production of fundamentally new and improved structural and other progressive materials accelerates the development of electronics, mechanical engineering, construction, the national economy, and other fields." Regarding construction, this primarily concerns structural materials—concrete and reinforced concrete—the use of which increases annually. This highlights the importance of the economic consumption of these materials, upon which the overall improvement of construction efficiency depends.

However, it is not only the pursuit of increased economic efficiency in the use of concrete and RC that dictates the necessity of perfecting their properties. Their scope of application is constantly expanding; accordingly, the requirements that concrete and RC must meet are increasing. This has led to intensive efforts, both domestically and abroad, to improve the economic indicators of using concrete and RC in various types of structures and operational conditions, as well as to find ways to endow them with properties that respond to new operational demands. These efforts are conducted in various directions, among which a significant role is played by the refinement of calculation methods for building structures. This allows for the identification of hidden reserves and the evaluation of reliability with a high degree of certainty, as well as the creation of traditional concrete types with improved indicators of durability, strength, and deformation.

Equally important are the works concerning the refinement of characteristics that determine the feasibility of using concrete and RC as structural materials, as well as more reliable methods for the quantitative determination of these characteristics.

Furthermore, significant importance should be assigned to studies whose task is to research the physical nature of the strength and deformation capacity of concrete and RC to reveal the mechanisms of resistance of these materials to the deforming and destructive impacts of structural and other factors. Specifically, from the standpoint of modern fracture mechanics of solids, there is great interest in studying the patterns of the formation and development of various types of structural cracks in concrete and RC. The strength and deformation capacity of concrete and RC—specifically the intensity of deformation development under constant loads, long-term strength in aggressive environments, the endurance limit, and similar issues—depend on the structural cracks formed and accumulated within the concrete.

Reinforced concrete is one of the most common and versatile materials in modern structural engineering. It is characterized by high strength, long service life, and a variety of forms. RC slab-type elements are widely used in both roofing and foundation structures. The formation of cracks in an RC slab is usually a manifestation of nonlinear mechanical processes. This process can be caused by a complex set of factors, such as excessively high applied loads, plastic deformation of the material, concrete shrinkage, creep and relaxation, thermal expansion and contraction, uneven impact on supports, or a non-homogeneous structure.

One of the most crucial issues determining the deformation and crack resistance of RC structures is the nature of reinforced concrete itself. As a two-component structural material, it differs from other materials by a range of characteristic properties. For example, properties such as heterogeneity, anisotropy, and crack initiation create certain difficulties in the calculation and design of building structures.

Regarding the novelty of the research, a study of cracks caused by the nonlinear deformations of RC slabs will be developed based on the analysis of research results. We will study the methodology, the deformed state of concrete and RC, and the process of crack formation, focusing on the patterns of crack initiation and development, as well as the quantitative assessment of their magnitudes based on the reinforcement ratio and the type of aggregate.

Thus far, during the construction of calculation equations, we have only formally touched upon the physical side of the problem. However, for fabricated slabs such as those made of reinforced concrete, substantiating the physical essence of the basic relationships is important not only for algorithmization and the construction of calculation models but also for understanding the phenomena occurring in RC structures under structural and deformational impacts. This ensures the exclusion of gross errors during the analysis of the nonlinear behavior of structures.

Normal cracks in longitudinal and transverse ribs, depending on the nature of the stress state, are reinforced either with distributed reinforcement or with reinforcing bars. The appearance of these cracks is associated with the action of No. 1 tensile forces.

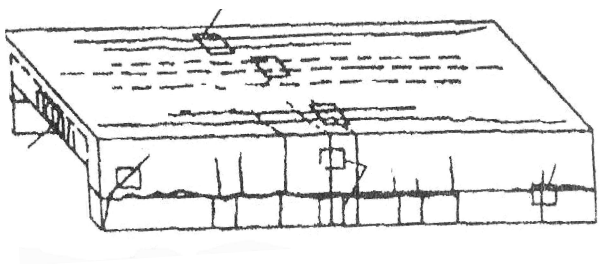

Fig. 1. Schematic diagram of crack formation

Inclined cracks near the supports of longitudinal ribs and in the corner zones of the shell are caused by principal tensile stresses on inclined planes. Depending on the nature of the stress state, the cracks in these zones are oriented orthogonally.

Two types of cracks may appear on the surface of the plates:

Longitudinal cracks at the junctions of the flange and the ribs.

Longitudinal cracks in the middle part of the flange span.

Longitudinal cracks along the connecting elements: their formation is associated with the action of shear forces S between the elements of the composite structure near the supports. The initiation and subsequent opening of these cracks are resisted by the bonds in the shear joint. Such bonds may include transverse reinforcement, dowel-type, or adhesive joints. The presence of transverse rod-connections in the contact zone of the element layers creates additional shear stiffness in the joint.

Composite reinforced concrete shells and corrugated panels are characterized by longitudinal curvature. The types of cracks are as follows:

Longitudinal cracks on the upper and lower surfaces of the flange, associated with the action of transverse bending moments.

Cracks in the end ribs, associated with the action of longitudinal tensile forces in the corresponding zones of the ribs.

Cracks in the upper chords of longitudinal diaphragms, caused by longitudinal tensile forces. The locations of these cracks are determined by the sign of the bending moment in the upper chord of the diaphragm.

Inclined and diagonal cracks in the support and corner zones. In different types of slabs, this type of crack can have varying effects on their load-bearing capacity.

When analyzing the nature of crack formation in composite slab panels, the following can be noted: most of the cracks arising in these structures belong to the type that is orthogonal to the working reinforcement. Consequently, when studying the deformation of this class of structures, orthotropic physical models of reinforced concrete can be utilized. An exception is made for inclined and diagonal cracks, which must be considered based on an anisotropic model. However, as is well known, during the calculation process when resolving static indeterminacy, even quasi-orthotropic or even quasi-isotropic models may be used for these types of cracks.

As a result of investigating the nature of crack formation for the structures under consideration, the model presented in the work (for the first-level calculation scheme) is adopted as the physical model, with certain refinements regarding the evaluation of crack opening. Generally, for elements e1 and e2 operating under plane stress conditions, the relationship between deformations and stresses can be represented as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{vmatrix} \times \begin{vmatrix} N_x \\ N_y \\ N_{xy} \end{vmatrix} \qquad (1)$$

Or

$$[\bar{\varepsilon}] = [C_{ji}] \cdot \{\bar{P}\} \qquad (2)$$

For elements e3 and e4, which operate under mixed stress conditions, according to the adopted hypotheses, we can write

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_y \end{Bmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{12}^* \\ C_{12} & C_{22} & C_{23} & C_{22}^* \\ C_{13} & C_{23} & C_{33} & C_{23}^* \\ B_{12}^* & B_{22}^* & B_{23}^* & B_{22} \end{vmatrix} \times \begin{vmatrix} N_x \\ N_y \\ N_{xy} \\ M_y \end{vmatrix} \qquad (3)$$

In expressions (1) and (2), C_{ji} is the stiffness matrix, the determination of which depends on the type of cracks and the reinforcement of the elements.

It is easy to see that the values of the stiffness coefficients in the constructed governing equations are directly related to the types of cracks. For instance, the first group of coefficients, C_{ji} determines the cross-sectional stiffness of the shell panel elements under tension (compression) and shear. Accordingly, this group of coefficients is associated with the first and second types of cracks. The coefficients in this group are calculated by multiplying the corresponding diagrams (stress/strain profiles) of the unit functions. For the first-level calculation scheme, the relationships of the deformational model of a quasi-continuous body are used, according to which the contribution of the tensioned concrete between the cracks is accounted for by introducing the coefficient ψ_{bt}

Let us examine the sequence of operations for calculating the first group of stiffness coefficients using shell panels as an example.

After the initiation of the first type of normal cracks, the zones of crack propagation on the structure's surface are determined in accordance with the given calculation scheme of the cross-section and the distribution diagrams of longitudinal forces in this section obtained during the current iteration of the calculation. To achieve this, the shell panel is divided along its length into m sections (m = 1, 2, ..., k), and within each section, the relative deformations of the tensioned concrete across the height of the cross-section are calculated using the following formula:

$$\varepsilon_{bt}(x,s) = \left[\sum_{c} \bar{U}_c'(x)\bar{\xi}(x) - \sum_{i} V_i''(x)\xi_i(s) \right] \quad (4)$$

For each calculation element, based on the obtained values of $\varepsilon_{bt}(x,s)$ (i=1, 2, ..., n), a distribution diagram $\varepsilon_{bt}(x)$ is constructed along the height of the structure's cross-section. The maximum deformation value, εbt,max, is then determined for the current iteration of the calculation. The condition for checking the initiation of a crack within the calculation element is written as follows:

$$\varepsilon_{bt,\max} \le \varepsilon_{btu} \quad (5)$$

Where ε_{btu} is the ultimate tensile strain value of the concrete. The numerical values of these strains can be determined as follows:

$$\varepsilon_{btu} = \frac{0,75 R_{bt,ser}\gamma_{b4}}{E_b v_{btu}} \quad (6)$$

Where

$$v_{btu} = v_{btR}\left[1 - 1,05 v_{btR}\sqrt{0m46 - 0,35 v_{btR}}\right]$$

$$v_{btR} = 0,6 + 0,15\frac{\gamma_{b4} R_{bt,ser}}{2,5} \quad (7)$$

For the elements in which cracks have appeared, the values of the $\psi_{bt,t}(s)$ coefficients are determined by the formula:

$$\psi_{bt,t}(s) = \frac{N_1(s)}{N_{crc}}(1-\psi_s) \quad (8)$$

where N_{crc} is the generalized crack formation force in the cross-section of the element in the direction of the principal tensile forces. The force N_{crc} can be determined by the general formula (9), if for the rib under consideration we take $\alpha = 85°$ and $m_{sy} = 0$

$$N_{crc} = \frac{\gamma_{b4} R_{bt,ser}}{v_{btu}} \left[t + 2\left(\alpha_{sx} m_{sx} \sin^2 \alpha + \alpha_{sy} m_{sy} \cos^2 \alpha \right) \right] \quad (9)$$

t is the thickness of the corresponding element. m_{xx}, m_{xy} – are the reinforcement parameters along the x and y axes. a_{sx}, a_{sy} – are the ratios of the elastic moduli of reinforcement and concrete along the x and y axes. α is the inclination angle of the principal plane, which is generally determined by the formulas for an anisotropic element. m_{sx}, m_{sy}, α_{sx}, α_{xy} are determined by the following expression:

$$m_{sx} = \frac{A_{sx}}{S_x} \quad m_{sy} = \frac{A_{sy}}{S_y} \quad \alpha_{sx} = \frac{E_{sx}}{E_0} \quad \alpha_{sy} = \frac{E_{sy}}{E_0} \quad (10)$$

where S_x and S_y are the distances between the reinforcement bars relative to the x and y axes.

The value of the edge thickness (taking cracks into account) at the i -th height of the section (i = 1,2,..., n) is determined from the expression:

$$t_i(s) = t_r \psi_{bt,i}(s) \quad (11)$$

The values of the stiffness coefficients $\overline{I}_{dc}(x,c)$ for (d, c = 0,1) for elements with cracks are calculated by the formulas:

$$\overline{I}_{00}(x,c) = 2\left[E_{b,1} t_r (h_1/2)(\psi_{bt,1} + \psi_{bt,2}) + E_s A_s \right]$$

$$\overline{I}_{01}(x,c) = 2\left[E_{b,1} t_r (h_1/2)(\psi_{bt,1} + \psi_{bt,2}) + E_s A_s \right]$$

$$\overline{I}_{11}(x,c) = 2\Big[E_{b,1} t_r (h_1/2)(\psi_{bt,1} + \psi_{bt,2}) + E_{b,2} t_r \left((z - h_1)/2 \right) \times$$
$$\times \left(\psi_{bt,3} + \psi_{bt,4} \right) + E_{b,2} t_r b_{b,1} \left(\overline{h} - z - t_p \right) + E_{b,2} t_{p,2} (t_b/2 + b) + E_s A_s \Big] \quad (12)$$

Calculation of the second group coefficients of the main diagonal $I_{ji}(x,s)$ $(j,i = 1)$ after the appearance of normal cracks in the edges of the structure; and the appearance of the crack is taken into account in the calculation by introducing the reduced thickness of the edges.

$$I_{11}(x,s) = 2\Big\{ E_{b,1} t_r \psi_{bt,1} (h_1/2)(z^2 - z h_1/2 + h_1^2/12) + + E_{b,1} t_r \psi_{bt,2} (h_1/2)(z^2 - 3 z h_1/2 + 7 h_1^2/12) +$$
$$+ E_{b,2} (7 t_r/3) \psi_{bt,3} \left((z - h_1)/2 \right)^3 + E_{b,2} (t_r/3) \psi_{bt,4} \left((z - h_1)/2 \right)^3 +$$
$$+ E_{b,2} (t_r/3) v_{b,1} \left((\overline{h} - z - t_p)/2 \right)^3 + E_{b,2} (t_p^2/2) v_{b,2} \times \times \left[\left(\overline{h} - z \right)^2 + \left(t_p/2 \right) \left(\left(z - \overline{h} + t_p \right)/2 \right) \right] +$$
$$+ E_{b,2} t_b v_{b,2} (\overline{h} - z)^2 + E_s A_s (z - a)^2 \Big\} \quad (13)$$

The determination of the equivalent reduced thickness of the edges (t_i) is performed using expression (11). The group of $I_{\mu}^{*}(x,s)$ coefficients is calculated by introducing the $\psi_{bti}(s)$ coefficient.

$$I_p^*(x,s)=2\left\{E_{b,1}t_r\left(h_1/2\right)\left[\psi_{bt,1}\left(z-h_1/4\right)+\psi_{bt,2}\left((z-3h_1)/4\right)\right]+\right.$$
$$+E_{b,2}(t_r/8)(z-h_1)^2(3\psi_{bt,3}-\psi_{bt,4})-E_{b,2}(t_r/2)v_{bm1}\left((\bar{h}-z-t_p)/2\right)-$$
$$\left.-E_{b,2}t_bv_{b2}\left[(t_p/2)\left((\bar{h}-z-t_p)/4\right)+b(\bar{h}-z)\right]+E_xA_s(z-a)\right\} \quad (14)$$

The group of $\alpha_{ji}(x,s)$, $S_{ji}(x,s)$ coefficients represents the longitudinal sectional flexural stiffnesses of the shell panel. Their values are determined by the reduced stiffnesses of longitudinal strips of unit width, which are cut from the shell panel by sections x and x+1. These stiffnesses are related to the formation of longitudinal cracks. The calculation of this group of coefficients is performed by multiplying the $f_1(s)$ diagram by its second-order derivative $f_1''(s)$.

$$\alpha_{11}(x,s)=2vE_{b,2}\Big/\left(12(1-v^2)\right)\left\{0,035b\left(t^3_{p,red.1}\right)^3\times\right.$$
$$\times\left[f_1(0,787b)\left(4f_1''(0,787b)+f_1''(b)\right)+f_1''(0,787b)+f_1(0,578b)\right]-$$
$$-0,048b\left(t^3_{p,red.1}\right)^3\left[f_1''(0,289b)(3f_1(0,578b)+4f_1(0,289b)+\right.$$
$$\left.\left.+f_1(0))+f_1''(0,289b)+2f_1(0))\right]\right\} \quad (15)$$

$$S_{11}(x,s)=2E_{b,2}\Big/\left(12(1-v^2)\right)\left\{0,096b\left(t^3_{p,red.1}\right)^3\times\right.$$
$$\times\left[2\left(f_1''(0,289b)\right)+\left(f_1''(0)\right)^2+f_1''(0,289b)f''(0)\right]+$$
$$\left.+0,08b\left(t^3_{p,red.2}\right)^3\left[2(f_1''(0,787b))^2+\left(f_1''(b)\right)^2+f_1''(0,787b)f_1''(b)\right]\right\} \quad (16)$$

Here $t^3_{p,red.j}$ is the reduced equivalent thickness of the flange in the j -th section within the zone of action of a transverse bending moment of a single sign, with width $dj(j$ = 1,2,..,) which is determined by the equivalent flexural stiffness.

$$t^3_{p,red.j}=\sqrt[3]{12B^3_{red,j}} \quad (17)$$

Where $B^3_{red,j}$ is the equivalent reduced stiffness of the longitudinal sections of the shell within the boundaries of the j-th segment. When determining the stiffness of the longitudinal sections and, consequently, the reduced sections of the shell $\left(t_{p,red}(s)\right)$ the reduction in the area of the concrete in tension due to cracking within each segment is the same as that used for calculating the first group coefficients—i.e., it is determined using the y_{bt} coefficient.

Conclusion: The study presents slabs with a freely supported contour and a crack located at the center. There are certain slabs whose geometry, loading, and yield-line patterns approximate cyclic symmetry; in such cases, calculations based on the theory of elasticity yield identical bending moments in both directions. In addition to circular and square slabs, this category may include regular polygonal slabs, slightly elongated rectangular slabs, etc., when subjected to uniform loading.

To determine the ultimate load-bearing capacity (intensity) of such slabs, an equation must be formulated equating the internal and external work done during a virtual displacement. The values for the work of external and internal forces within the formula are calculated by assigning a unit translational velocity to the instantaneous center of intersection of the plastic hinges.

When extending the yield lines, they intersect at the center of the crack. The diagram of angular velocities is constructed according to the principle of Cremona diagrams. Additionally, the study covers the investigation of cracks caused by non-linear deformations in reinforced concrete slabs, specifically cases where the crack is situated between the diagonals of the slab.

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