Determination of internal forces in sections of curved arcuate rod at its deformation

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DOI: https://doi.org/10.52340/building.2025.71.13

Abstract. The article discusses the calculation of the strength of a prestressed curved rod in the form of a round arch by the method of displacement. Such arches are used in building structures and in various machine units, and need to be comprehensively studied by methods of resistance of materials. This problem is a fundamental problem of building mechanics of machines and various building stationary structures.
Keywords: rod, curvature, displacement, force, bending moment.

Introduction

Some parts of various building structures and some machine units are under the influence of forces applied from interaction with various adjacent parts. These forces cause the stress-strain state of these parts.

As is known in order to increase the strength of machine-building and building structures, some of their parts are given a curvilinear shape, which significantly increases their rigidity and carrying capacity. In addition, the use of such parts makes it possible to create these structures as light as possible, and to produce less material consumption during their manufacture.

Main part

A circular arch of constant cross-section, which at point D has a rigid fastening, is a statically detectable elastic system. After pre-stretching its end point B and fixing it to the fixed post C, the curved element becomes pre-stressed and it will already be statically indeterminate twice.

The curved rod of circular shape is stretched by Δ_x along the axis Ox and is connected to the fixed hinge C. It is necessary to find the bending moment, transverse force and longitudinal force in an arbitrary section of the rod.

The problem is statically indeterminate, based on this, to determine the value of the X_1 and X_2, it is necessary to draw up two equations of displacement of the end of the rod, i.e. Canonical elastic curvilinear rod deformation equations.

$$\left.\begin{aligned}\Delta_{F1} + \delta_{11}X_1 + \delta_{12}X_2 = \Delta_x \\ \Delta_{F2} + \delta_{21}X_1 + \delta_{22}X_2 = 0\end{aligned}\right\} \tag{1}$$

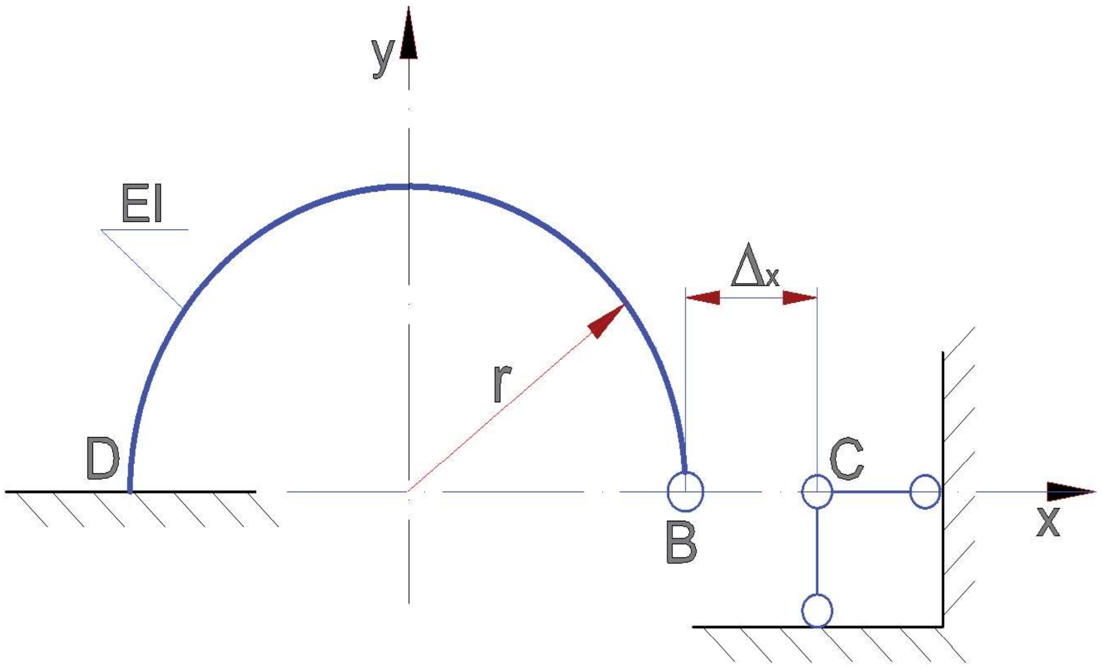

Fig. 1

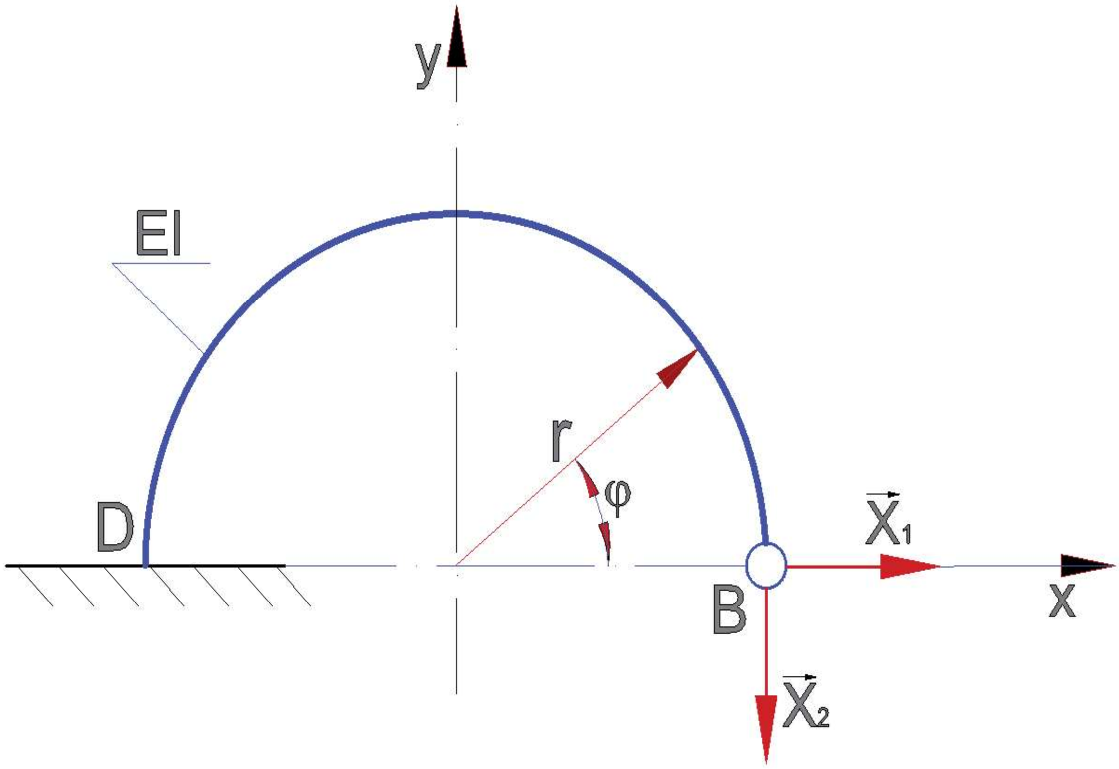

Fig. 2

Given that:

$$M_F = 0, \quad M_1 = r \cdot sin\varphi, \quad M_2 = -r \cdot (1 - cos\varphi), \qquad (2)$$

We find:

$$\Delta_{F1} = \Delta_{F1} = 0, \quad (3)$$

Based on this, the coefficients in the system of canonical equations are determined by the following expressions:

$$\delta_{11} = \int_0^{\pi} \frac{(r \cdot sin\varphi)^2 \cdot r}{EI} d\varphi = \frac{\pi r^3}{2EI} \quad (4)$$

$$\delta_{22} = \int_0^{\pi} \frac{r^3(1-cos\varphi)^2}{EI} d\varphi = \frac{3\pi r^3}{2EI}, \quad (5)$$

$$\delta_{12} = \delta_{21} = -\int_0^{\pi} \frac{r^3 \cdot sin\varphi \cdot (1-cos\varphi)^2}{EI} d\varphi = -\frac{2r^3}{EI}, \quad (6)$$

After substituting the obtained values of these coefficients into the system of canonical equations, we obtain expressions that determine the values of unknown forces acting at the end of the arch:

$$X_1 = \frac{6\pi EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3}, \quad (7)$$

$$X_2 = \frac{8EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3}, \quad (8)$$

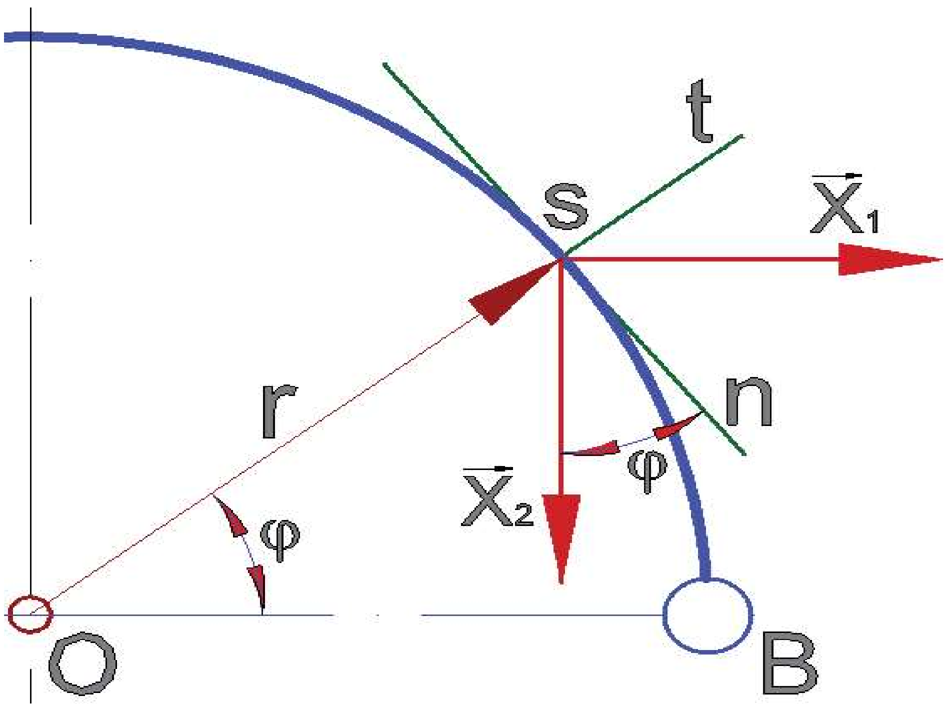

Fig. 3

Knowing the values of these forces, you can determine the internal forces in any section of the arch (Fig. 3).

The bending moment in any section of the arch will be:

$$M = X_1 \cdot r \cdot sin\varphi - X_2 \cdot r(1 - cos\varphi) = \frac{6\pi EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot r \cdot sin\varphi - \frac{8EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot r \cdot (1 - cos\varphi) =$$

$$= \frac{2EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^2} \cdot [3\pi \cdot sin\varphi - 4 \cdot (1 - cos\varphi)] \quad (9)$$

The transverse force in any section of the arch is given by:

$$Q = X_1 \cdot cos\varphi - X_2 \cdot sin\varphi = = \frac{6\pi EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot cos\varphi - \frac{8EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot sin\varphi = = \frac{2EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot [3\pi \cdot cos\varphi - 4 \cdot sin\varphi] \quad (10)$$

The longitudinal force in any section of the arch will be:

$$N = X_1 \cdot sin\varphi + X_2 \cdot cos\varphi = = \frac{6\pi EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot sin\varphi + \frac{8EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot cos\varphi = = \frac{2EI}{3\pi^2-16} \cdot \frac{\Delta_x}{r^3} \cdot [3\pi \cdot sin\varphi + 4 \cdot cos\varphi] \quad (11)$$

Conclusion

Some machine installations and building structures contain curved parts that work in conditions of force interaction with other parts of the structure. Such interactions can create deformations of these elements, which in turn can cause the occurrence of additional normal and tangential stresses, which can exceed the permissible values, and this can cause the part to collapse and disable the entire unit. Calculations for the strength and rigidity of such parts are an urgent task of modern mechanics and one of the solutions to this problem is shown in this work.

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