Analysis of plastic torsion using physical analogies

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Abstract calculating the torsional deformations of a complex section is a rather complex task even in the elastic stages. Additional difficulties arise when it is necessary to calculate rods of complex cross-section at the stage of plasticity. To at least partially get rid of these difficulties, this work uses a method widely known as the method of analogies. We specifically use a branch of this method that we call physical analogies. In particular, the so-called membrane analogy and hydrodynamic analogy. The problem of torsion of a homogeneous shaft is discussed. The latter is based on the phenomenon that the process of torsion of a solid rod (no matter what cross-section it has) is mathematically expressed by the same formulas as the flow of an ideal fluid.

Key words: membrane analogy, hydrodynamic analogy, rod of constant cross-section, plastic torsion, shear stress.

1. Introduction

In our work [4] were considered the difficulties that accompany the calculation of complex cross-section rods for torsional deformation in the elastic stage. The aim of this paper is to calculate rods also of complex cross-section for torsion, in the plastic stage, which is associated with additional difficulties. Let us turn first to the analogy with a membrane, according to which at the initial stage the membrane moves in a plane (x, y). Let's influance on it by a little pressure p. As a result, the membrane deflects slightly so that the tensile strain per unit length S becomes constant, i.e. the membrane contour will remain in the plane $z = 0$. It's equation of balance will be

$$S\frac{\partial^2 z}{\partial x^2} + S\frac{\partial^2 z}{\partial y^2} + p = 0; \qquad \nabla^2 z = -\frac{p}{S} \qquad (1)$$

This equation is similar to the equation given in our paper [4].

where, ψ - stress function, G - shear modulus, θ - relative shear angle. We can exploit this similarity in the following way: ψ – are identified with transverse displacements z of membrane points, and $2G\theta$ - with relation p/S. On this basis, if the contour of the membrane is the same as the cross section of the rod, then the surface of this membrane as a result of the pressure being in equilibrium determines the stress function for the rod cross section. Griffith and Taylor developed apparatus that is now widely used for practical experiments.

2. Main part

As an example, consider the problem about torsion of a homogeneous shaft (Fig. 1). According to formula (1), a flat annular membrane connected to a circular frame with radius a, under transverse pressure p stretches and takes the shape of a paraboloid segment. The sum of projections of forces acting on the membrane on the vertical axis will be

$$p\pi r^2 = -S \cdot 2\pi r\frac{\partial r}{\partial z}; \quad \frac{\partial z}{\partial r} = -\frac{rp}{2S} \qquad (3)$$

where S - power. acting per unit length. Accordingly $z = -\frac{pr^2}{4S} + C$.

If $z = 0$, $r = a$, then $z = (a^2 - r^2)$.

so,

$$\psi = (a^2 - r^2)G\frac{\theta}{2} \qquad (4)$$

Since the maximum tangential stress at the point $|grad\psi|$, i. e. $-\frac{d\psi}{dr}$, then $\tau = G\theta r$.

τ_{max} is going to come up when $r = a$ and respectively $\tau_{max} = G \cdot \theta \cdot a$

Volume between the plane and the membrane $V = \int_A z \cdot dA = \int_0^a \frac{P}{4S}(a^2 - r^2)2\pi r dr = \frac{P \cdot \pi \cdot a^4}{8S}$. (5)

Torque

$$T = 2V = G\theta\pi \cdot \frac{a^4}{2} = G\theta I,$$

where $I = \frac{\pi a^4}{2}$ - polar moment of inertia about the central axis.

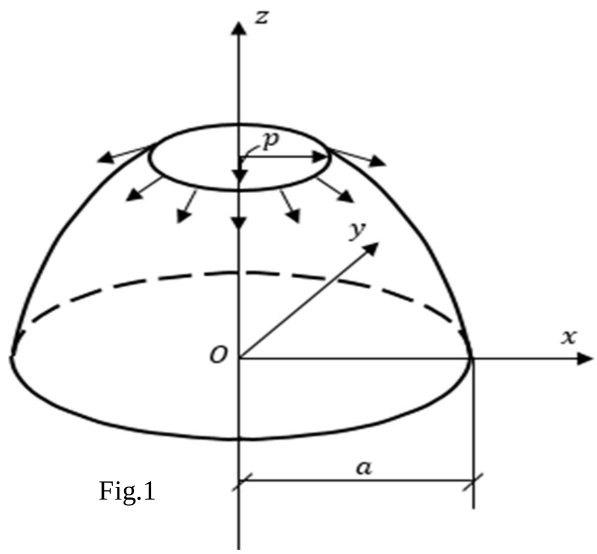

Fig.1

In many cases, the maximum tangential stress occurs at the point on the perimeter of the rod cross-section closest to the central axis. However, this regularity is broken for cross sections of some shapes.

The next analogy used in the study of torsion is known as the hydrodynamic analogy. It is based on the assumption that the torsion of a rod of constant cross-section is mathematically described by the same formulas as the flow of an ideal fluid flowing through a pipe of the same cross-section with constant angular velocity. At that, the fluid circulation velocity at some point corresponds to the tangential torsional stress at the same point. There is an analogy between the cylindrical torsion of a rod and the potential of a plane fluidity field. It is used to study the concentration of tangential stresses at characteristic points (e. g. edges). Let us now turn to those analogies which, according to our observations, give better results in cases of calculations in the plastic stage of torsion. As stated, $|grad\psi|$ denotes the total tangential stress developed at the point. Its physical interpretation would be the maximum slope of the membrane surface at the same point. let τ_{fl} will be the stress corresponding to the yield strength, i.e. under conditions of plastic deformation $|grad\psi| = \tau_{\text{fl}}$ and hence, the value that the gradient takes at the specified point is bounded. As the torque increases further, the area over which the tangential stresses reach the yield strength increases. In (Fig. 2) the increase of plastic flow areas depending on the increase of the torsion angle of the rod is shown. For a section of equilateral triangle shape (shaded areas), according to this analogy, plastic deformations tend to appear near the surface and spread inward toward the central axis. At plastic deformation of areas adjacent to the outer surface, the tangential stresses are equal to the yield stress at shear, to which is corresponded the constancy of the membrane slope.

For elastic deformation regions, the tangential stresses from the value σ_{fl} at the boundaries of the plastic and elastic regions decrease to zero on the axis (meaning an ideally plastic material). Accordingly, the value of the membrane slope changes. Since the tangential stresses are continuous at these boundaries, in the plastic deformation region

$$\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2 = \tau_{fl}^2 \quad (6)$$

and in the region of elastic deformations

$$\left(\frac{\partial\psi}{\partial x}\right)^2 + \left(\frac{\partial\psi}{\partial x}\right)^2 = -2G\theta \quad (6')$$

The use of the membrane analogy in the analysis of elastic-plastic flow faces great difficulties, and in the case of ideally plastic flow, the work of calculating the torque is significantly simplified, since the membrane is completely in contact with the curved surface and the entire section is covered by plastic deformations (there is no longer an elastic core).

The problem is solved more easily when using the so-called sand analogy, which is as follows: dry sand is poured onto a plate, which has the cross-sectional shape of a prismatic rod, which maintains a constant angle of inclination. The use of this analogy is especially useful when the cross-sectional shape is too complex to be described mathematically.

We use this technique to find the torque that causes plastic deformation of the entire section for the following sections: a) circular b) equilateral triangle c) rectangle

a) circular cross-section with radius a, slope $\frac{h}{a} = \tau_{\text{fl}}$, volume $\frac{1}{3}\pi \cdot a^2 \cdot h$. Torque

$$T = \frac{2}{3}\pi \cdot a^3 \cdot \tau_{\text{fl}} \quad (7)$$

b) an equilateral triangle with side $2a$: slope $\frac{h}{\frac{1}{3}a\sqrt{3}} = \tau_{fl}$. Torque

$$T = \frac{2a^3\tau_{\text{d}}}{3} \quad (8)$$

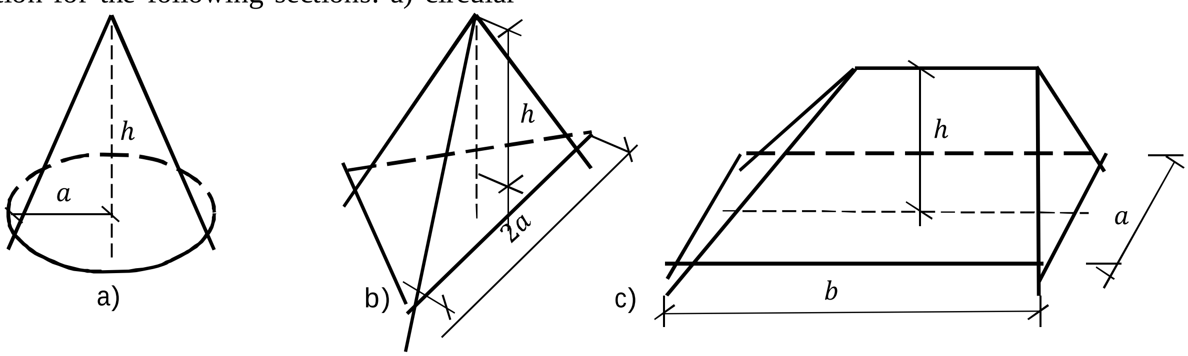

Fig. 2

c) a rectangle with sides a и b: slope: $\frac{h}{\frac{a}{2}} = \tau_{fl}$, The volume is equal to $(b-a)\frac{ah}{2} + 2\left(\frac{1}{3}\cdot\frac{a^2h}{2}\right)$,

Torque

$$T = a^2\tau_{fl}\frac{(3b-a)}{6} \quad (9)$$

In case of square

$$T = \frac{a^3\tau_{fl}}{3} \quad (10)$$

As can be seen from what has been discussed, when using the sand analogy method, you always have to calculate the volume of sand. The more complex the cross-sectional shape, the more difficult this task becomes. Therefore, in the scientific, expert, experimental laboratory of the Faculty of Civil Engineering of the Georgian Technical University, we replaced the complex process of calculating volume with the process of weighing sand, which greatly simplified the solution of the tasks and we obtained accurate results, of course, as a result of comparison. Using the sand analogy method led us to the following conclusions that in some cases its use is limited for the following reasons:

1) Metals used in practice undergo partial hardening during torsional deformation, so plastic deformations do not completely cover their cross sections.

2) The cross sections of the rods change their shape before plastic deformation covers the entire section, i.e. we have both elastic and plastic deformations at the same time. Therefore, ignoring changes in this form affects the accuracy of the calculation.

3) Due to side effects, the length of the rod changes during the torsion process.

Despite the above, in the absence of a better method, analogies with sand are used in practice, since, in spite of everything, they make it possible to quantify the work of metals at the plastic torsion.

Conclusion

A number of problems of torsion of homogeneous rods of complex cross-section at the stage of plastic deformation, interesting for small and medium-sized fields of mechanical engineering, have been studied. The so-called method of physical analogies was used. Expressions of torques and stresses for specific cases were obtained. A comparative analysis of the phenomena of elastic-plastic and ideal plastic flow is carried out. The difference characterizing these two processes is shown.

It is shown for the first time that the problems of ideal plastic flow presented in the article can be solved in a simpler way than is achieved when considering the phenomenon of elastic-plastic flow.

Reference

1. Malinin N.N. Applied Theory of Plasticity and Creep "Machine Building", M.1975 p. 399 (Russian)
2. Hill R. Mathematical Theory of Plasticity. GITTL, 1956, p. 407 (Russian)
3. T. Batsikadze, N. Murgulia, J. Nizharadze "Derivation of Equations of Flow Theory Using Plastic Potentials". Scientific and Technical Journal "Construction". No. 4 (64), 2022. (Russian)
4. Tamaz Batsikadze, Nugzar Murgulia, Jumber Nizharadze „Use of plastic potential in the theory of fluidity" Scientific-technical journal „Building". #4(68), 2023.