Calculation of horseshoe-shaped lining in elastic environment

Demur Tabatadze, David jankarashvili, Ioseb Kakutashvili
Georgian Technical University, Tbilisi, Georgia, 77, M. Kostava St. 0160
d.tabatadze@gtu.ge

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Abstract The horseshoe-shaped lining in an elastic medium is deformed both from the inside and from the outside. From the outside, the lining receives elastic resistance from the soil, which changes along a square parabola and helps the lining work. Taking into account the influence of elastic resistance of the rock on the work of the tunnel lining is of qualitative importance. The paper considers the calculation of a horseshoe-shaped lining in an elastic medium, taking into account the influence of elastic resistance of the rock. The system under consideration is statically indeterminate, the calculation is carried out using the method of forces in matrix form. Examples of calculation are given.

Key Word: Horseshoe-shaped lining; elastic medium; soil resistance; method of forces in matrix form

Introduction

Calculation of the vault taking into account elastic resistance is a relatively complex task. Experiments have established that the law of distribution of the resistance diagram can be approximated by a square parabola. The vault is a static indefinite system, the calculation of which can be made by the method of forces in matrix form.

Calculation of tunnel lining

This calculation takes into account the influence of elastic rock resistance [1]. When calculating, the following are specified: the shape of the resistance diagram and its zone of action, and the greatest value q^1_{max} the resistance intensity is determined under the assumption that the rock is an elastic Winkler base. In addition to the rock resistance, the influence of friction forces is also taken into account, the intensity of which at each point of the outer surface of the lining is equal to the rock resistance intensity at this point, multiplied by the friction coefficient f (t^1=f· q^1). The selection forces are directed along the normal, and the friction forces are tangential to the outer surface of the lining.

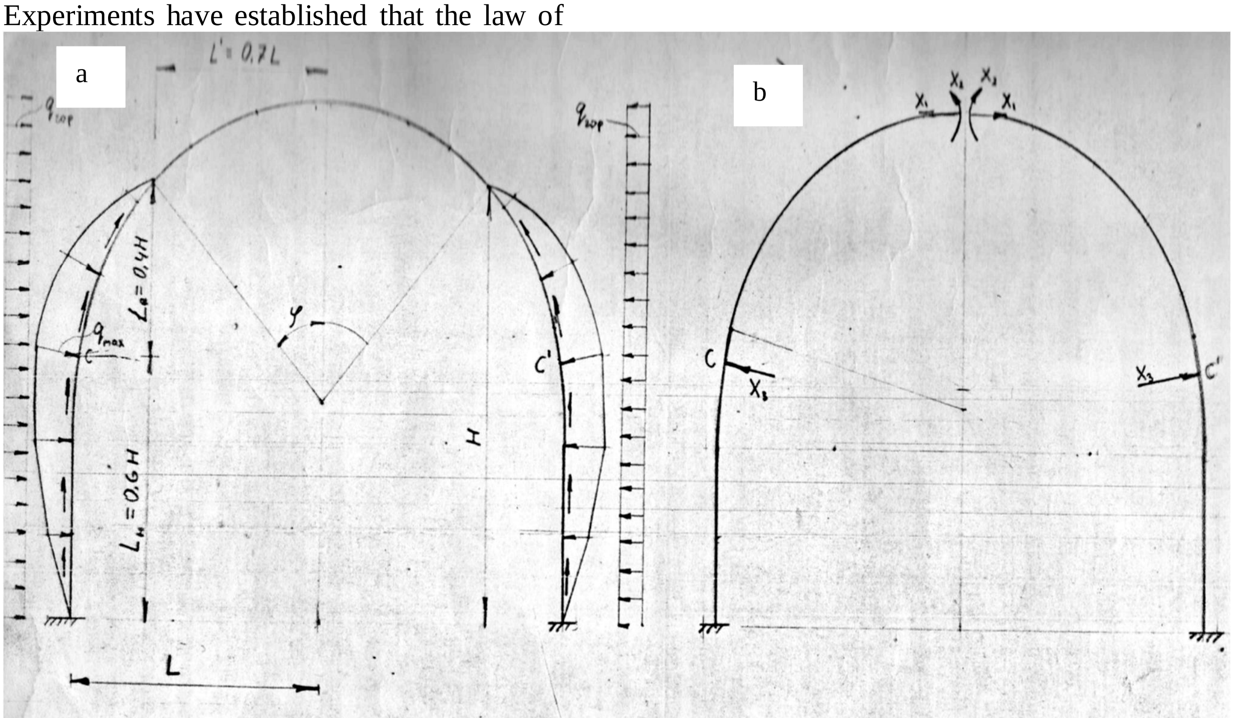

Figure (1a) shows the load diagram, and Figure (1b) shows the design diagram of the arch

The unknowns are the efforts X_i and the magnitude of the intensity q^1_{max}. They are found from the canonical equations of the force method and the equation

$$q^1_{max} = k_c \cdot v^1_{max} \quad (1.1)$$

where k_c is the coefficient of rock bedding on the sides of the arch (т/м^3), V^1_{max} – displacement of that point of the lining at which the resistance of the rock has the greatest value ordinate of the diagram q_{max}. The largest ordinate of the resistance diagram q^1_{max} is accepted as being located at the level (0.33-0.4)H, where H defines the zone of action of elastic selection and is established graphically from Fig. 1. The distribution law q^1 are set as follows:

$$q^1 = q^1_{max}\left(1 - \frac{Z^2}{L^2}\right), \quad (1.2)$$

where Z is the vertical distance from point C at which $q^1 = q_{max}$. It is assumed that point C is removed from zero point A by a distance of 0.4 H. When constructing the diagram q^1 for the section of the arch located above point C, instead of L,

$$L_B = 0.4\ \text{H},$$

And below this point $- \text{L} = L_н = 0.6\text{H}.$

The canonical equations of the force method are:

$$\delta_{11}X_1 + \delta_{12}X_1 + \delta_{1q^1} \quad + \Delta_{1P} = 0$$
$$\delta_{21}X_1 + \delta_{22}X_1 + \delta_{2q^1} \quad + \Delta_{2P} = 0 \quad (1.3)$$

In these equations the coefficients δ_{11}, δ_{12}, δ_{21}, δ_{22} and free members Δ_{1P}, Δ_{2P} have the usual meaning, as for the coefficients δ_{1q^1} и δ_{2q^1}, then they represent a movement in the direction of unknowns X_1 и X_2, caused by the action of a single rock rebuff q^1_{max}=1 and the corresponding friction forces $t^1 = q^1 \cdot f$. Let's apply at point C and C[1] forces directed along the normal to the geometric axis of the arch X_3. Let us denote by δ_{31}, δ_{32} and δ_{3q^1} displacement in the direction of force X_3 from single impacts: $X_2 = 1$, X_2 =1 and single selection of the breed, and through , Δ_{3P} – from a given external load. Then the total displacement in the direction of the dash from a given external load. Then the total displacement in the direction of the dash X_3 equals:

$$\Delta_3 = V^1_{max} = \delta_{31}X_1 + \delta_{32}X_1 + \delta_{3q^1} \quad + \Delta_{3P}$$

Based on the formula (1.1), $V^1_{max} = \frac{1}{K_C} q^1_{max}$. Taking the latter into account, we have:

$$\delta_{31}X_1 + \delta_{32}X_1 + \left(\delta_{3q^1} - \frac{1}{K_C}\right) q^1_{max} + \Delta_{3P} = 0 \quad (1.4)$$

Equations (1.3) and (1.4) can be represented in matrix form

$$\delta \cdot X + \Delta = 0 \quad (1.5)$$

where $\delta = \begin{Vmatrix} \delta_{11} & \delta_{12} & \delta_{1q^1} \\ \delta_{21} & \delta_{22} & \delta_{2q^1} \\ \delta_{31} & \delta_{32} & \left(\delta_{3q^1} - \frac{1}{K_C}\right) \end{Vmatrix}$;

$$X = \begin{Vmatrix} X_1 \\ X_2 \\ X_3 \end{Vmatrix}; \qquad \Delta = \begin{Vmatrix} \Delta_{1P} \\ \Delta_{2P} \\ \Delta_{3P} \end{Vmatrix}; \qquad (1.6)$$

The matrix can be obtained by the formula:

$$\delta = \overline{S^T}\, F\, \overline{S_1} + K$$

Where $\bar{S}$ - matrix of unit efforts. In the first column of the matrix $\bar{S}$ located efforts in the main system from the force: X_1 =1, in the second column from X_2 =1 and in the third from - X_3 =1. S_1 - is also a matrix of unit efforts. It differs from the matrix $\bar{S}$ the fact that in its third column there is no effort in the main system, from a single resistance of the rock, that in its third column there is no effort in the main system, from X_3 =1, and from a single resistance of the rock (при q^1_{max}=1).

$$K=\begin{Vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{K_C} \end{Vmatrix} \qquad (1.8)$$

The matrix Δ can be obtained by the formula:

$$\overline{S^T}\, F\, S_P \qquad (1.9)$$

Here S_P - effort matrix.

The final forces in the static indeterminate tunnel lining are determined by the formula

$$S = (\overline{S^1_1}\, X + S^1_P)P = (-\overline{S^1_1} \cdot \delta^{-1} + S^1_P)P \qquad (1.10)$$

Where δ и Δ matrices are determined by the formula (1.7) и (1.9). $\overline{S^1_1}$ − unit effort matrix. It is different from the Matrix $\overline{S_1}$ in that it includes efforts, the value of which is determined by q^1_{max}=1.

Example 1

Based on the above method, the arch is calculated. The vertical and horizontal loads are respectively equal to: $q_{верт.}=25\ \frac{т}{м^2}$, $q_{гор.}=1.5\ \frac{т}{м^2}$.

The coefficient of rock bedding on the sides of the arch is equal to $K_П = 16\ \frac{т}{см^3} = 16 \cdot 10^3 \frac{т}{м^3}$.

All geometric dimensions of the arch are given in Table No. 1. We begin the calculation by determining the internal forces in the main rock support system. ($q_{верт.}=1\ \frac{т}{м^2}$) и от external specified load (see table No. 1).

$$S=\begin{vmatrix} 0 & 1 & 0 \\ 0.23 & 1 & 0 \\ 0.89 & 1 & 0 \\ 1.93 & 1 & 0 \\ 3.27 & 1 & 0 \\ 4.70 & 1 & 0 \\ 6.30 & 1 & 1.63 \\ 7.03 & 1 & 3.29 \\ 9.55 & 1 & 4.94 \\ 0 & 0 & -0.19 \\ 9.55 & 1 & 4.94 \end{vmatrix}$$

$$S_1=\begin{vmatrix} 0 & 1 & 0 \\ 0.23 & 1 & 0 \\ 0.89 & 1 & 0 \\ 1.93 & 1 & -0 \\ 3.27 & 1 & -1\ .02 \\ 4.70 & 1 & -4.16 \\ 6.30 & 1 & -9.69 \\ 7.03 & 1 & -17.65 \\ 9.55 & 1 & -26.89 \\ 0 & 0 & -26.89 \\ 9.55 & 1 & -26.89 \end{vmatrix}$$

$$S_P=\begin{vmatrix} 0 \\ -32.19 \\ -119.17 \\ -236.07 \\ -349.43 \\ -428.41 \\ -463.42 \\ -465.09 \\ -502.05 \\ 147.25 \\ -502.05 \end{vmatrix}$$

$$\overline{S}^1_1=\begin{vmatrix} 0 & 1 & 0 \\ 0.23 & 1 & 0 \\ 0.89 & 1 & 0 \\ 1.93 & 1 & -0.02 \\ 3.27 & 1 & -1.02 \\ 4.70 & 1 & -4.16 \\ 6.30 & 1 & -9.69 \\ 7.03 & 1 & -17.65 \\ 9.55 & 1 & 26.89 \\ 1 & 0 & 0 \\ 0.96 & 0 & 0 \\ 0.89 & 0 & 0 \\ 0.67 & 0 & -0.023 \\ 0.44 & 0 & -0.15 \\ 0.19 & 0 & -0.12 \\ 0 & 0 & 0.06 \\ 0 & 0 & -0.29 \\ 0 & 0 & -0.43 \end{vmatrix};$$

#	corner			Coordinates in m			Section thickness	Efforts in the main system			
								From resistance and friction forces q^1_{max}=1		From the specified load	
	φ	φ sin	φ cos	X	Y	Y_1	h	$M_{T.M.}$	N_T	$M_{T.M.}$	N_T
0	0	0	1	0	0	0.23	0.70	0	0	0	0
1	0.27	0.27	0.96	1.60	0.23	0.89	0.75	0	0	-32.19	10.74
2	0.56	0.53	0.84	3.08	0.89	1.93	0.775	0	0	-119.17	-39.73
3	0.83	0.74	0.66	4.323	1.93	3.27	0.825	-0.02	-0.023	-236.07	78.42
4	1.12	0.89	0.43	5.23	3.27	3.27	0.875	-1.02	-0.154	-349.93	115.48
5	1.38	0.98	0.19	5.74	4.70	4.70	0.95	-4.16	-0.124	-428.41	139.45
6	1.57	1	0	5.89	6.30	6.30	1.00	-9.69	-0.055	-463.42	147.25
7	1.57	1	0	5.89	7.93	7.93	1.20	-17.65	-0.294	-465.09	147.25
8	1.57	1	0	5.89	9.55	9.55	1.26	-26.89	-0.428	-502.05	147.25

$$S_P^1=\begin{vmatrix} 0 \\ -32.19 \\ -119.17 \\ -236.07 \\ -349.93 \\ -428.41 \\ -463.42 \\ -465.09 \\ -502.05 \\ 0 \\ 10.74 \\ 39.73 \\ 78.42 \\ 115.48 \\ 139.55 \\ 147.25 \\ 147.25 \\ 147.25 \end{vmatrix}$$

F=
|28.43; 46.22; 41.09; 34.10; 28.61; 22.74; 19.50; 11.28; 9.74; 17.14; 119.05|

$$\overline{S_1^1}=\begin{vmatrix} 0 & 10.63 & 36.57 & 65.81 & 93.55 & 106.88 & 122.85 & 89.45 & 93.02 & 0 & 1336.93 \\ 28.43 & 46.22 & 41.09 & 34.10 & 28.61 & 22.74 & 19.5 & 11.28 & 9.74 & 0 & 119.05 \\ 0 & 0 & 0 & 0 & 0 & 0 & 31.79 & 37.11 & 48.12 & -3.26 & 588.11 \end{vmatrix}$$

$$(\bar{S}^T\cdot F)\cdot\overline{S_1}=\begin{vmatrix} 141.39 & 1745.11 & -30572.05 \\ 1745.11 & 360.76 & -3975.67 \\ 6570.46 & 705.13 & -18219.87 \end{vmatrix};$$

$$\delta=(\bar{S}^T\cdot F)\cdot\overline{S_1}+K=\begin{vmatrix} 141.39 & 1745.11 & -30572.05 \\ 1745.11 & 360.76 & -3975.67 \\ 6570.46 & 705.13 & -18219.87 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{24.103}{16\cdot 10^3} \end{vmatrix};$$

$$K=\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{24.103}{16\cdot 10^3} \end{vmatrix};$$

$$\Delta = (\bar{S}^T \cdot F) S_P = \begin{vmatrix} -814790.25 \\ -113129.97 \\ -351890.92 \end{vmatrix}; \quad X = \begin{vmatrix} 53.94 \\ 94.49 \\ 3.79 \end{vmatrix}; \quad S_1^1 \cdot X = \begin{vmatrix} 94.49 \\ 106.89 \\ 142.99 \\ 198.51 \\ 267.00 \\ 332.21 \\ 397.53 \\ 455.25 \\ 507.5 \\ 53.94 \\ 51.78 \\ 36.04 \\ 23.16 \\ 9.79 \\ 0.23 \\ -1.10 \\ -1.63 \end{vmatrix}; \quad S = \begin{vmatrix} 94.49 \\ 74.70 \\ 23.32 \\ -37.56 \\ 82.93 \\ -96.20 \\ -65.89 \\ -9.84 \\ 5.52 \\ 53.94 \\ 62.52 \\ 85.58 \\ 114.47 \\ 138.64 \\ 149.34 \\ 146.15 \\ 145.62 \end{vmatrix}$$

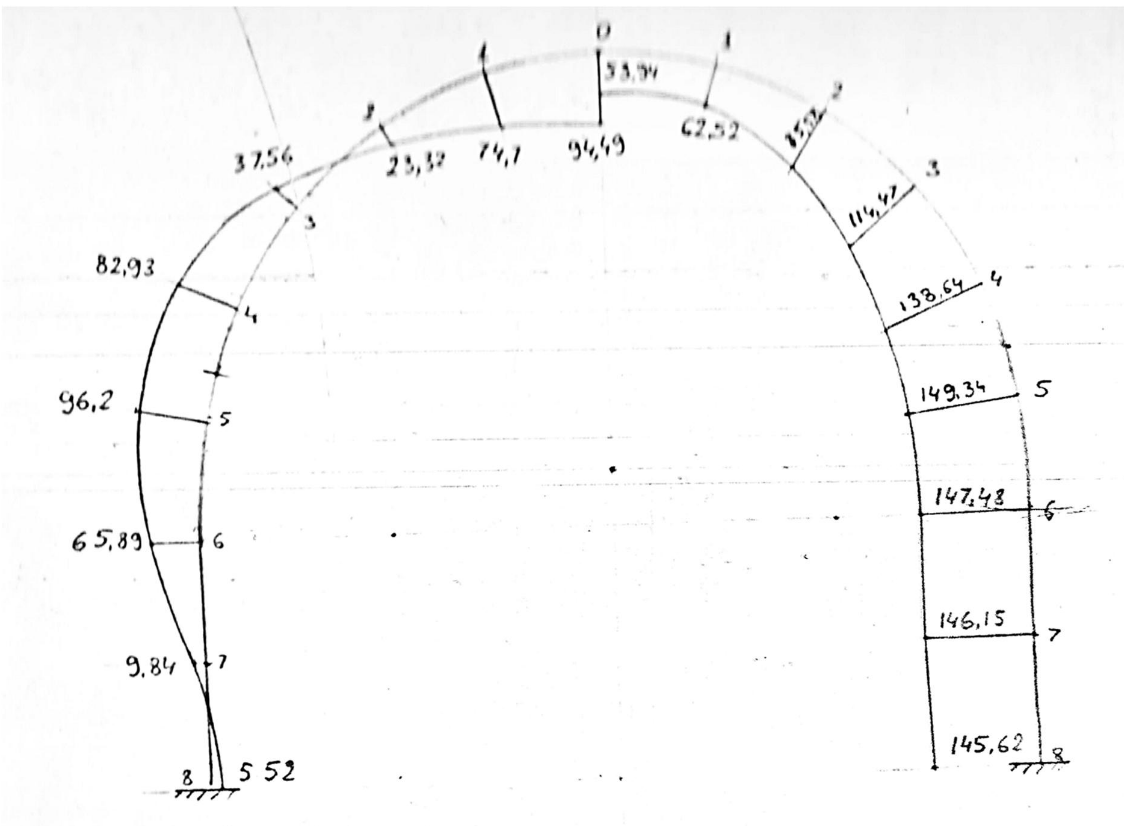

Figure 2.

The arch is calculated on the vertical – $q_{верт}$=25 $\frac{т}{м^2}$ and on the horizontal $q_{гор}$=1.5 $\frac{т}{м^2}$ loads. The coefficient of rock bedding on the sides of the arch is equal to $K_П$=1.10^3 $\frac{т}{м^2}$. loads. The coefficient of rock bedding on the sides of the arch is equal to

$$K = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{24.103}{16 \cdot 10^3} \end{vmatrix};$$

$$\delta=(\bar{S}^T\cdot F)\cdot\bar{S_1}+K=\begin{vmatrix}14199.57 & 1745.11 & -30572.05\\ 1745.11 & 360.76 & -3975.67\\ 6570.46 & 705.13 & -20469.87\end{vmatrix};$$

$$\Delta=(\bar{S}^T\cdot F)S_P=\begin{vmatrix}-814790.25\\ -113129.97\\ -351890.92\end{vmatrix};\quad X=\begin{vmatrix}51.34\\ 92.52\\ 2.48\end{vmatrix};$$

$$S=\begin{vmatrix}92.52\\ 72.14\\ 19.04\\ -44.51\\ -92.05\\ -104.88\\ -71.44\\ -9.13\\ 14.20\\ 51.34\\ 60.03\\ 83.37\\ 111.74\\ 137.70\\ 149.00\\ 147.40\\ 146.53\\ 146.19\end{vmatrix}$$

The diagrams of the bending moment and longitudinal force are shown in Fig. 3.

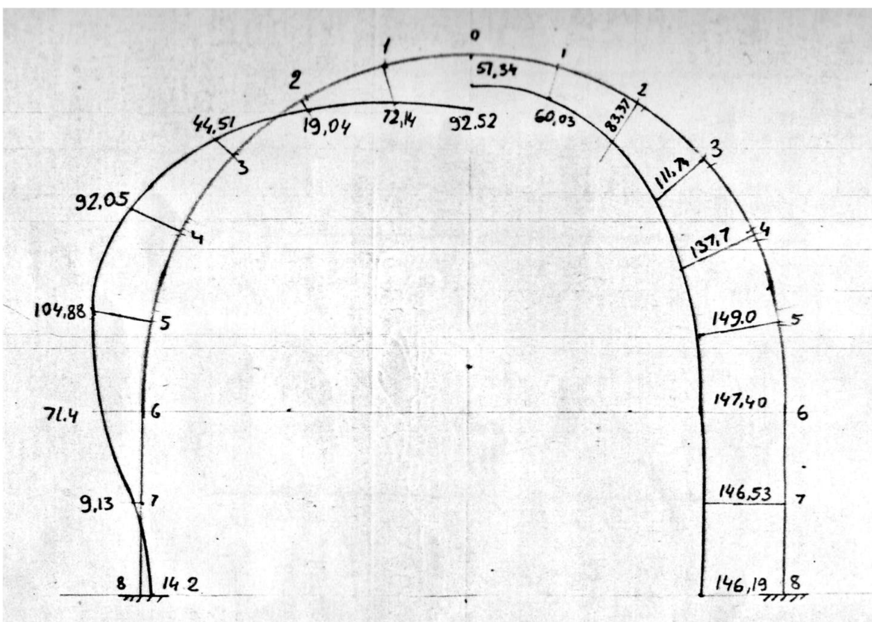

Figure 3.

The arch is calculated for the same loads and geometry without taking into account the influence of the elastic foundation. The corresponding matrices in this case have the form:

$$X=\begin{vmatrix}46.47\\88.82\end{vmatrix};\quad \delta=(\bar{S}^T\cdot F)\cdot\bar{S_1}=\begin{vmatrix}14199.57 & 1745.11\\1745.11 & 360.76\end{vmatrix};\quad \bar{S_1}=\begin{vmatrix}-814790.25\\-113129.97\end{vmatrix};$$

$$S=\begin{vmatrix}88.52\\67.32\\11.00\\-57.57\\-92.05\\-109.17\\-121.20\\-81.87\\-7.80\\30.52\\57.21\\55.34\\79.23\\109.55\\135.92\\148.38\\147.25\\147.25\end{vmatrix};$$

The plots are shown in Fig. 4.

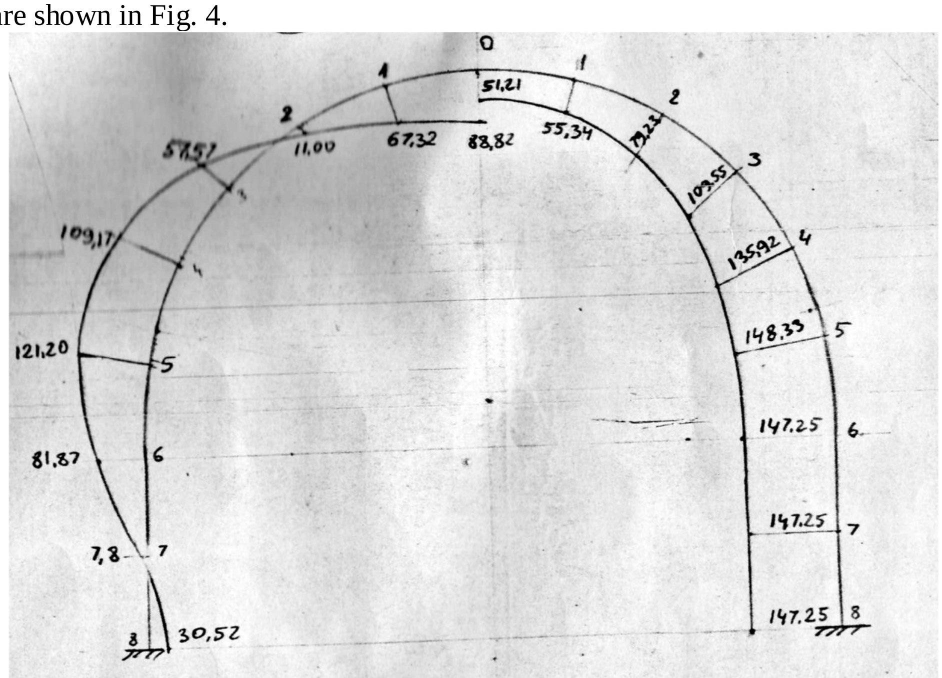

Figure 4

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