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THE STATIONARY PROBLEM OF THE BOUNDARY LAYER FORMED BY THE JOINT ROTATION OF A POROUS CIRCULAR PLATE AND THE SUROUNDING CONDUCTIVE FLUID TAKING INTO ACCOUNT THE MAGNETIC FIELD AND HEAT TRANSFER IN THE CASE OF VARIABLE SUCTION VELOSITY

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Abstract. In the paper, the stationary joint rotation of a porous circular plate and the surrounding conductive fluid is studied by the Shvetz method (method of successive approximation), taking into account the magnetic field and heat transfer, when the suction velocity changes according to the linear law.

In order to solve the problem, the Navier-Stokes differential equations of fluid motion in a magnetic field and energy nonlinear differential equations in partial derivatives by using generalized Karman transformations are reduced to ordinary nonlinear differential equations, the solutions of which are sought in the form of infinite series. The first two approximations have been clearly calculated, which determine the distribution of fluid velocity, temperature and pressure in the dynamic and thermal boundary layers formed on the circular plate.

In order to calculate the thicknesses of the dynamic and thermal boundary layers, the appropriate equations are obtained and the exact solutions of these equations are recorded. In a particular case, the relationship between the thicknesses of the dynamic and thermal boundary layers is determined. The moment of resistance to rotation of the plate and the heat transfer coefficient are also calculated.

Key words: Conductivity; injection velocity; flow; heat transfer; magnetic field; porosity.

Introduction.

The differential equations of viscous fluid motion and the energy equation were derived in the middle of the XIX century. As is known,

The fact that the effect of viscosity should be felt in the vicinity of the surface of the enveloping body was indicated as early as by problem of the boundary layer formed by the they represent the mathematical formulation of the laws of conservation of mass, momentum and energy for an elementary volume of a liquid or gaseous environment. These equations are the basis for the theoretical analysis of problems related to fluid dynamics, heat transfer, subsonic and supersonic flows, and many other flows. The considered equations are second-order private derivative nonlinear differential equations, which determines significant difficulties in their solution.

Until the beginning of the XX century, the theoretical solution of many practically important problems of heat transfer and hydrodynamics was difficult. This is explained by the fact that the Navier-Stokes differential equations of fluid motion and the energy equation cannot be analytically solved.

In the XIX century, some problems were solved in the special case when the inertial forces in the Navier-Stokes equations are equal to zero. One of these tasks is to determine the hydraulic resistance during laminar flow of liquid in a pipe.

Using Euler's equations (equations in which the action of viscous forces is not taken into account), the velocity field in the vicinity of the surrounding body can be calculated, as well as the pressure forces on the surface of the body can be determined. However, the ideal fluid theory cannot explain the reason for the emergence of vortices in the rear part of poorly enveloping bodies. In the case of the transverse girdle of the cylinder, this leads to Dalember's paradox: due to the symmetrical distribution of pressure around the surface of the cylinder, the resistance force is zero.

D. Mendeleev (1880) in his studies, which were devoted to the study of the resistance generated during the movement of bodies in a liquid.

Experiments and important theoretical considerations indicate that in some cases fluid motion is significantly affected by the fact that the fluid does not slide only immediately near the boundary in the thin layer covering the surface of the enclosing body. In this regard, the theory of the boundary layer, inside which the viscosity cannot be neglected, and which makes sense for large values of the Reynolds number, was born.

Boundary layer theory is one of the important parts of modern hydromechanics. This theory was founded in 1904 by by L. Prandtl [1] in a paper presented at the International Congress of Mathematicians in Heidelberg. Prandtl formulated the equations, which are satisfied in the first approximation by the speed of fluid movement in the boundary layer. These equations are called the Prandtl system. They form the basis of the boundary layer theory, which has been intensively developed for more than a century.

Prandtl divided fluid flow into two regions: one, inside the boundary layer, where viscosity plays an essential role and most of the resistance experienced by the surrounding body, and the other, outside the boundary layer, where viscosity can be neglected without significantly affecting the fluid. This idea makes it possible to significantly simplify the system of Navier-Stokes equations. Much of the heat transfer to and from the body also occurs within the boundary layer, again simplifying the energy equation for the fluid flow field outside the boundary layer.

The works of Carman [2] and Cochrane [3] were devoted to the study of problems related to fluid flow caused by the rotation of an infinite plate or disk with a large radius. They used transformations (Kármán embeddings) that allow nonlinear partial differential equations of fluid motion to be written in the form of ordinary nonlinear differential equations, which makes it possible to study the nature of the flow more deeply.

From a practical point of view, it is interesting to study problems related to the movement of porous axisymmetric bodies surrounded by liquid. Stewart [4] was one of the first scientists who studied in his work the influence of the suction velocity of liquid from the surface of a porous circular disk on the flow of liquid generated by the rotation of the disk.

Many scientists are interested in the study of such stationary and non-stationary problems related to the flow of electrically conductive fluid in the boundary layer, when the influence of a weak or strong magnetic field and thermal effects are taken into account in the vicinity of the surfaces of the surrounding porous bodies.

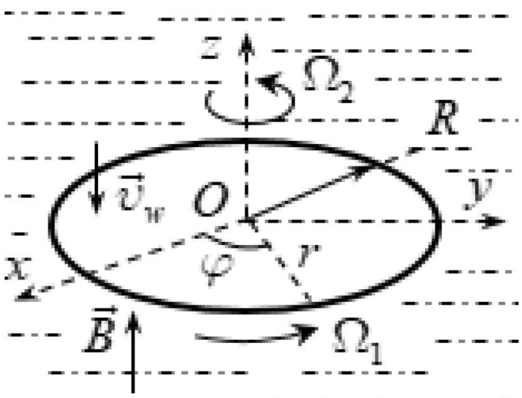
In the work [5], the non-stationary problem of the boundary layer formed by the rotation of a porous plate in a weakly stationary fluid is studied, taking into account the heat transfer, when the same fluid flows into the plate at a rate that is a time-dependent quantity. The thicknesses of the dynamic and thermal boundary layers for different values of the suction velosity are determined and the functional relationships between them are established.

In the paper [6], the non-stationary problem of the joint rotation of an infinite porous plate and the surrounding fluid is studied by the sequential approximation method, taking into account the magnetic field and heat transfer, when the coefficient of electrical conductivity and the suction velosity are temperature-dependent quantities.

In the work [7], the stationary problem of the boundary layer formed by the rotation of a porous circular plate in a conductive liquid is studied by the method of sequential approximation, taking into account a weak uniform magnetic field and heat transfer, when the fluid suction velosity in the plate changes according to a linear law.

Main part.

In the present work, the stationary task of the boundary layer formed by the joint rotation of a circular porous plate and the surrounding fluid is studied by the method of Shvets [8], taking into account the magnetic field and heat transfer, in the case of variable suction velosity.



Let's say a porous circular plate of radius R rotates about an Oz axis with an angular velocity Ω_1 in an electrically conducting fluid, which in turn rotates with an angular velocity Ω_2. Let's assume that the same liquid is leaking through the plate with a speed $\vec{u}_w$ varying according to the linear law, a uniform $\vec{B}$ magnetic field acts perpendicular to the plate, its temperature is T_w and the temperature of the liquid is T_∞ at infinity. The goal of the task is to study the fluid flow in the vicinity of the plate and determine all the physical and dynamic characteristics of the fluid.

Consider that at a long distance from the plate, where there is no friction [10], we have

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \Omega_2^2 r.$$

Then the system of Navier-Stokes differential equations and the energy equation of the motion of a conductive fluid in a magnetic field will be written in the following form

$$\begin{cases} v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\varphi^2}{r} = -\Omega_2^2 r + \nu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2}\right) - \frac{\sigma B^2}{\rho} v_r, \\ v_r \frac{\partial v_\varphi}{\partial r} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_r v_\varphi}{r} = \nu\left(\frac{\partial^2 v_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial v_\varphi}{\partial r} + \frac{\partial^2 v_\varphi}{\partial z^2} - \frac{v_\varphi}{r^2}\right) - \frac{\sigma B^2}{\rho}\left(v_\varphi - \Omega_2 r\right), \\ v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right), \\ \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0, \end{cases} \quad (1)$$

$$\rho c_p\left(v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z}\right) = \lambda\left(\frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2}\right). \quad (2)$$

In the energy equation, we consider that the influence of dissipative members on heat transfer is infinitesimally small. To solve the system of equations (1) and equation (2), we use the following boundary conditions:

$$\begin{cases} z = 0,\ v_r = 0,\ v_\varphi = \Omega_1 r,\ v_z = -v_w,\ T = T_w, \\ z = \infty,\ v_r = 0,\ v_\varphi = \Omega_2 r,\ T = T_\infty. \end{cases} \quad (3)$$

From mechanical considerations, if we introduce new functions and make the following transformations of variables

$$\begin{cases} v_r(r,z) = r\omega f(\eta),\ v_\varphi(r,z) = r\omega q(\eta),\ v_z(z) = \sqrt{\nu\omega}\, g(\eta), \\ v_w(z) = \sqrt{\nu\omega}\, u_w(\eta),\ p(z) = -\rho\nu\omega P(\eta), \\ \eta = \sqrt{\frac{\omega}{\nu}} z,\ T(z) = T_\infty + (T_w - T_\infty)\theta(\eta), \end{cases} \quad (4)$$

then, by means of them, (1) system of equations and (2) equation will be written in the following form:

$$\begin{cases} \frac{d^2 f}{d\eta^2} - m^2 f - g\frac{df}{d\eta} - f^2 + q^2 = 0, \\ \frac{d^2 q}{d\eta^2} - m^2 q - g\frac{dq}{d\eta} - 2fq = 0, \\ \frac{dP}{d\eta} = -\frac{d^2 g}{d\eta^2} + g\frac{dg}{d\eta}, \quad \frac{dg}{d\eta} + 2f = 0, \end{cases} \quad (5)$$

$$\frac{d^2\theta}{\partial\eta^2} = P_r\left(g\frac{d\theta}{d\eta}\right). \quad (6)$$

In the obtained equations, $m^2 = \frac{\sigma B^2}{\rho\omega}$ and $P_r = \frac{\mu c_p}{\lambda}$ are the magnetic interaction and Prandtl numbers. Let's say the suction velocity in the plate changes according to the following law:

$$u_w = -b_\eta$$

where b is a positive number.

In order to calculate the thicknesses of the dynamic and thermal boundary layers formed on the surface of the circular porous plate rotating in the liquid, instead of the asymptotic layers, the layers of finite thickness are considered. To determine their thickness, we use the following conditions:

$$\eta=\delta,\ \frac{dq}{d\eta}=0 \text{ and } \eta=\delta_T,\ \frac{d\theta}{d\eta}=0. \quad (7)$$

Finally, from all this, the system of differential equations (5) and equation (6) must be solved under the following boundary conditions:

$$\begin{cases}\eta=0,\ f=0,\ q=\omega_1,\ g=u_w(0)=0,\\ \qquad\qquad \theta=1,\\ \eta=\delta,\ f=0,\ q=\omega_2,\ \dfrac{dq}{d\eta}=0,\\ \eta=\delta_T,\ \theta=0,\ \dfrac{d\theta}{d\eta}=0.\end{cases} \quad (8)$$

Let's look for the solutions of system (5) and equation (6) in the form of infinite series

$$\begin{cases} f=\sum_{i=0}^{\infty} f_i, \quad q=\sum_{i=0}^{\infty} q_i,\\ g=\sum_{i=0}^{\infty} g_i, \quad \theta=\sum_{i=0}^{\infty} \theta_i,\end{cases} \quad (9)$$

and limit ourselves to defining the first two approximations.

If we calculate the first two approximations of the f, q, g and q functions, then we will have the following expressions for the fluid velocity components, temperature and pressure:

$$\upsilon_r=\frac{R^2}{\nu R_e}r(\Omega_2-\Omega_1)\left[-\frac{\Omega_2-\Omega_1}{12\delta^2}\eta^4-\frac{\Omega_1}{3\delta}\eta^3+\right.$$
$$\left.+\frac{\Omega_2+\Omega_1}{2}\eta^2-\frac{3\Omega_1+5\Omega_2}{12}\delta\eta\right],$$

$$\upsilon_\varphi=r\left\{\Omega_1+\frac{\Omega_2-\Omega_1}{\delta}\left[\frac{m^2-b}{6}(\eta^3-\delta^2\eta)-\right.\right.$$
$$\left.\left.-\frac{m^2\delta}{2}(\eta^2-\delta\eta)+\eta\right]\right\},$$

$$\upsilon_z=-\nu b\frac{\sqrt{R_e}}{R}\eta+\frac{(\Omega_2-\Omega_1)}{\nu}\left(\frac{R}{\sqrt{R_e}}\right)^3\times$$
$$\times\left[\frac{\Omega_2-\Omega_1}{30\delta^2}\eta^5+\frac{\Omega_1}{6\delta}\eta^4-\frac{\Omega_2+\Omega_1}{3}\eta^3+\right.$$
$$\left.+\frac{3\Omega_1+5\Omega_2}{12}\delta\eta^2\right],$$

$$T=T_\infty+\frac{T_w-T_\infty}{6\delta_T}\left[6\delta_T-6\eta+P_r b(\eta^3-\delta_T^2\eta)\right],$$

$$p=p_0-\frac{\rho\nu bR_e}{R^2}+\frac{\rho R^2(\Omega_2-\Omega_1)}{R_e}\left[\frac{\Omega_2-\Omega_1}{6\delta^2}\eta^4+\right.$$
$$\left.+\frac{2\Omega_1}{3\delta}\eta^3-(\Omega_2+\Omega_1)\eta^2+\frac{3\Omega_1+5\Omega_2}{6}\delta\eta\right]-$$
$$-\frac{\rho R^6}{2\nu R_e}\left\{-b\eta+\frac{\Omega_2-\Omega_1}{3}\left[\frac{\Omega_2-\Omega_1}{10\delta^2}\eta^5+\frac{\Omega_1}{2\delta}\eta^4-\right.\right.$$
$$\left.\left.-(\Omega_2+\Omega_1)\eta^3+\frac{3\Omega_1+5\Omega_2}{4}\delta\eta^2\right]\right\}^2.$$

To calculate the thicknesses of dynamic and thermal boundary layers, using the conditions (7), the following equations are obtained:

$$(m^2+2b)\delta^2-6=0, \quad bP_r\delta_T^2-3=0.$$

from where

$$\delta=\sqrt{\frac{6}{m^2+2b}}, \quad \delta_T=\sqrt{\frac{3}{bP_r}}.$$

In a private case, when $m=0$ and $P_r=1$, then $\delta=\delta_T=\sqrt{\frac{3}{b}}$.

If we calculate the moment of resistance of rotation of the plate and the coefficient of heat transfer, the following images are obtained:

$$M=-\frac{\pi\mu R^3\sqrt{R_e}}{12\delta}(\Omega_2-\Omega_1)\left[6+(2m^2+b)\right],$$

$$N=r\left(1-\frac{T_\infty}{T_w}\right)\left(\frac{1}{\delta_T}-\frac{P_r b}{6}\delta_T\right),$$

where $R_e=\frac{\omega R^2}{\nu}$ - Reynolds number.

Conclusion.

The sequential approximation method used to solve the problem under consideration by us allows us to define the search functions in any approximation. Also note that based on the obtained results, it is easy to see how the magnetic field, rotational angular velocities, suction parameter, Reynolds and Prandtls numbers and plate radius affect the physical characteristics of fluid flow and heat transfer.

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