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Selection of Mathematical Optimization Methods for Solving Engineering Practice Problems

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Abstract

Optimization theory plays a pivotal role in contemporary scientific and technical endeavors, permeating various engineering disciplines. From fine-tuning chemical-technological systems to optimizing production processes, the application of optimal management techniques is widespread, particularly in the context of complex automation and sophisticated technical setups. The primary goal of optimization is to identify the most optimal solution among numerous potential outcomes, employing diverse strategies ranging from analytical methodologies to numerical simulations. This paper explores the efficacy of the fastest ascent method in approaching the extremum of the Rosenbrock function, emphasizing the importance of selecting appropriate starting coordinates. Furthermore, the study investigates the impact of errors introduced through random variables, highlighting the need for robust methodologies capable of navigating uncertainties. Through comprehensive analysis and experimentation, this research contributes to the ongoing discourse surrounding optimization methodologies, shedding light on their effectiveness and applicability in diverse engineering contexts.

Key words: Optimization theory, engineering practice, optimal management techniques, fastest ascent method, Rosenbrock function.

Introduction

At the modern stage, the theory of optimization makes a significant contribution to scientific and technical progress. It is difficult to find a field of engineering activity where the problem of optimization in the tasks to be performed is not solved. It can be the task of determining the effective mode of operation of the chemical-technological system, the operation of various technical devices, the tasks of solving the problem of the production organization, and others.

Optimal management is widely used in conditions of complex automation of technological and industrial processes or complex technical equipment.

The goal of optimization is to select the best solution among potentially possible outcomes using appropriate performance criteria. Optimization can be carried out using many different strategies, which can begin with the use of complex analytical and quantitative procedures and end with the judicious use of simple arithmetic.

Numerical optimization is one of the central methods of machine learning. For many problems it is difficult to determine the best direct solution, but it is relatively easy to determine an error function that measures how accurately the chosen method is, and then the task of minimizing the parameters of this function in order to find the best solution.

Researchers and engineers are often faced with the challenge of predicting the behavior of certain systems or processes in order to control them. This task can be solved through mathematical models [1,2] and numerical modeling [3]. Although numerical simulations usually provide a good prediction of the behavior and properties of a certain system [3], initially, it is impossible to determine which of the many alternatives is the best choice [2].

Since the research activity is aimed at finding an alternative with the best properties, engineers and researchers in the field of engineering optimization actively use a mathematical approach, optimal control methods [3]. In engineering practice, it often happens like this - the goal of optimization is mathematically determined by the objective function - which is formulated taking into account technical or economic requirements , which is based on trials and research, which allows us to get the system with the best data. However, when using a scientific approach to solve a real problem, we are faced with an infinite number of optimization methods and corresponding software for the formulation and solution of the optimization problem. Since there is no universal optimization algorithm that can be used to solve any problem, it is important to evaluate the effectiveness of these methods in different conditions.

Main Part

Our primary objective is to explore the most efficient access method, especially when the bid length is contingent upon the characteristics of the optimization function. The swiftest approach involves an iterative algorithm that navigates towards the extremum within the specified range of argument values. This process entails moving from a chosen point towards the direction of the function's minimum value. This direction is essentially the opposite of that indicated by the gradient vector (∇f(x)) of the optimization function f(x).

$$\nabla f(x) = \left[\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots, \frac{\partial f}{\partial x_n}\right]^T, \qquad (1)$$

The formula for determining the argument x_{k+1} with the value x_k on the *k*-th bit using the fastest approach method is as follows: $x_{k+1} = x_k + \lambda_k \cdot S_k$

where S_k is a unit vector pointing in the opposite direction of the gradient $\nabla f(x)$at the specified point x_k.

$$S^{(k)} = -\frac{\nabla f(x^{(k)})}{\|f(x^{(k)})\|}$$

We tested the algorithm using the minimization of a function of two variables, specifically the Rosenbrock function, as an example. We established the necessary parameters, with the notations aligning with those used in the Mathcad software system:

n:=20 - the maximum number of iterations on the x and y axes;

i: 0..n, j: 0n - the sequence number of calculations;

a1:=−0.2, a2:=−0.2;

b1:=0.06, b2:=0.06;

xi=a1+b1·i, yj=a2+b2·j;

Formulas for computing the arguments of the i-th and j-th order:

Mij:=f(xi,yj) - matrix of Rosenbrock function values.

As depicted in the figure, the function exhibits a flat bottom. Near this bottom, the gradient assumes small values. Hence, it's apparent from the outset that an algorithm unaffected by the function's shape will be less efficient in approaching the minimum.

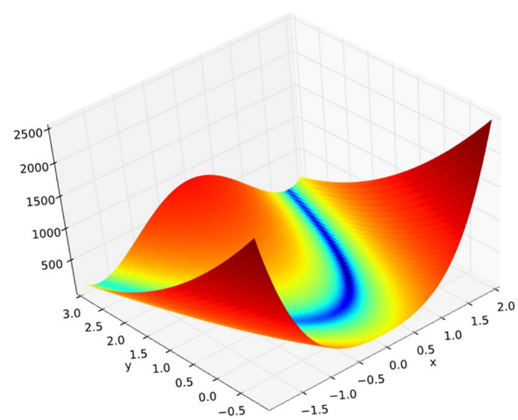


Figure 2 Graphic representation of the Rosenbrock function

Now, let's proceed with the computation of the elements of the gradient vector for the Rosenbrock function:

a) Partial derivatives with respect to the arguments x and y: g_x(X,Y), g_y(X,Y);

b) Second-order partial derivatives with respect to x and y: gxx(X,Y), gyy(X,Y); c) Mixed second-order derivatives with respect to x and y g_xy(X,Y), g_yx(X,Y).

The following parameters are necessary for implementing the fastest ascent method algorithm:

vmax:=200 - the maximum number of iterations required for approaching the minimum;

V:=0…vmax - the range of iteration changes;

x0:=0,y0:=0 - the coordinates of the starting approach point;

λ0:=λ_f(x0,y0) - the value corresponding to the initial approximation;

sx0:=s_x(x0,y0), sy0:=s_y(x0,y0) - the values of the bit corresponding to the initial approximation;

f0:=ff(x0,y0) - the value of the optimization function corresponding to the initial approximation.

Vector of initial values for the iterative process:

$$\begin{bmatrix} x_0 \\ y_0 \\ f_0 \end{bmatrix} := \begin{vmatrix} x0 \\ y0 \\ f0 \end{vmatrix}$$

Please provide the image of the function coordinates calculation and their corresponding values for further analysis.

$$\begin{bmatrix} x_{v+1} \\ y_{v+1} \\ ff_{v+1} \end{bmatrix} = \begin{bmatrix} xv + \lambda\_f(xv, yv).s\_x(xv, yv) \\ yv + \lambda\_f(xv, yv).s\_y(xv, yv) \\ ff(xv + \lambda\_f(xv, yv).s\_x(xv, yv), +\lambda\_f(xv, yv).s\_y(xv, yv)) \end{bmatrix}$$

x =

	0
236	0.93900212
237	0.93921365
238	0.9393041
239	0.93951411
240	0.93960314
241	0.93981165
242	0.9398993
243	0.94010632
244	0.94019262
245	0.94039818
246	0.94048316
247	0.94068728
248	0.94077096
249	0.94097366
250	0.94105607
251	0.94125737

y =

	0
236	0.88185501
237	0.88182281
238	0.88242049
239	0.88238872
240	0.88298067
241	0.88294931
242	0.88353562
243	0.88350468
244	0.88408546
245	0.88405492
246	0.88463026
247	0.88460012
248	0.88517011
249	0.88514036
250	0.8857051
251	0.88567573

ff =

	0
237	$70394858 \cdot 10^{-3}$
238	$68563878 \cdot 10^{-3}$
239	$3.667426 \cdot 10^{-3}$
240	$3.6493836 \cdot 10^{-3}$
241	$63143631 \cdot 10^{-3}$
242	$61365553 \cdot 10^{-3}$
243	$59596798 \cdot 10^{-3}$
244	$3.5784432 \cdot 10^{-3}$
245	$56100985 \cdot 10^{-3}$
246	$54373563 \cdot 10^{-3}$
247	$52655106 \cdot 10^{-3}$
248	$50952212 \cdot 10^{-3}$
249	$49258112 \cdot 10^{-3}$
250	$47579233 \cdot 10^{-3}$
251	$3.4590898 \cdot 10^{-3}$
252	

Using the obtained values, we constructed a diagram (Fig. 2). Upon examining the values, it becomes evident that there is oscillation near the minimum.

The diagram illustrates the dependency of the function coordinates on the iteration number as the extremum is approached.

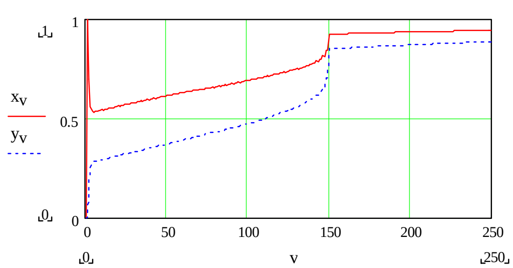


Figure 3 Dependence of function coordinates on iteration order for approximation to extremum

We carried out research for different initial values.

a) x0:=1.5 y0:=2 - coordinates of the starting point of approach;

Table 1 fragment of the table of obtained values:

x =

	0
236	1.33074929
237	1.33001466
238	1.33033907
239	1.32960748
240	1.32993078
241	1.32920219
242	1.3295244
243	1.32879877
244	1.32911991
245	1.32839721
246	1.32871728
247	1.32799747
248	1.32831648
249	1.32759953
250	1.3279175
251	1.32720337

y =

	0
0	2
1	2.02390156
2	2.02550898
3	2.01787204
4	2.01734204
5	2.01055437
6	2.01004465
7	2.00390547
8	2.00341208
9	1.99778778
10	1.99730798
11	1.99210448
12	1.99163623
13	1.98678442
14	1.98632617
15	1.98177356

ff =

	0
0	6.5
1	0.20481523
2	0.17888568
3	0.17766212
4	0.17646521
5	0.17537835
6	0.17431231
7	0.17332953
8	0.17236365
9	0.17146333
10	0.1705771
11	0.16974406
12	0.16892302
13	0.16814611
14	0.16737961
15	0.16665039

Table 1 A fragment of the table of obtained values:

As evident from the graph, the approach to the extremum with starting points x0=1.5 and y0 =2 is notably coarse, even under ideal conditions. Thus, we sought comparisons with other initial coordinates.

Subsequently, we conducted the research under the specified initial conditions (fig.3 and fig.4), accounting for errors introduced by random variables following a normal distribution:

g:=rnorm(130,0,0.33).

Incorporating these errors, the iterative procedure takes the following form:

$$\begin{bmatrix} x_{(v+1)} \\ y_{(v+1)} \\ ff_{(v+1)} \end{bmatrix} \begin{matrix} x_v + \lambda\_f(x_{v,}y_v).\, s\_x(x_v, y_v) + (x_v + \lambda\_f(x_{v,}y_v).\, s\_x(x_v, y_v)).\, pr.\, gv \\ y_v + \lambda\_f(x_{v,}y_v).\, s\_y(x_v, y_v) + (y_v + \lambda\_f(x_{v,}y_v).\, s\_y(x_v, y_v)).\, pr.\, gv \\ (x_v + \lambda\_f(x_{v,}y_v).\, s\_x(x_v, y_v) + (x_v + \lambda\_f(x_{v,}y_v).\, s\_x(x_v, y_v)).\, pr.\, gv \\ y_v + \lambda\_f(x_{v,}y_v).\, s\_y(x_v, y_v) + (y_v + \lambda\_f(x_{v,}y_v).\, s\_y(x_v, y_v)).\, pr.\, gv) \end{matrix}$$

x =

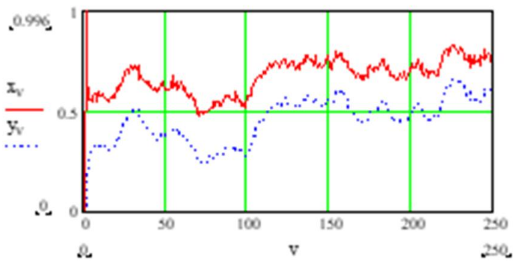
	0
86	0.568
87	0.578
88	0.57
89	0.564
90	0.559
91	0.555
92	0.542
93	0.589
94	0.567
95	0.56
96	0.528
97	0.544
98	0.518
99	0.531
100	0.573
101	0.575

y =

	0
86	0.304
87	0.321
88	0.325
89	0.32
90	0.314
91	0.309
92	0.3
93	0.319
94	0.325
95	0.317
96	0.295
97	0.29
98	0.28
99	0.277
100	0.303
101	0.321

ff =

	0
236	0.056
237	0.203
238	0.164
239	0.098
240	0.068
241	0.079
242	0.097
243	0.123
244	0.059
245	0.177
246	0.081
247	0.062
248	0.049
249	0.112
250	0.065
251	0.135



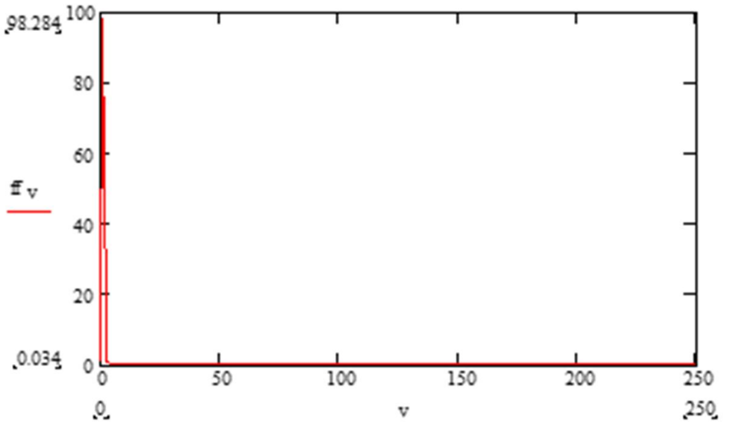


Figure 4 Fragment of the lookup table and corresponding diagrams x0:=0; y0:=0; ξ=10%;

x =

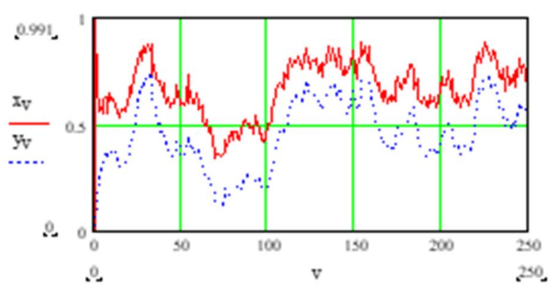
	0
86	0.504
87	0.526
88	0.512
89	0.501
90	0.491
91	0.483
92	0.46
93	0.539
94	0.502
95	0.488
96	0.432
97	0.457
98	0.413
99	0.433
100	0.502
101	0.509

y =

	0
86	0.23
87	0.258
88	0.264
89	0.255
90	0.245
91	0.236
92	0.221
93	0.248
94	0.258
95	0.245
96	0.211
97	0.201
98	0.187
99	0.18
100	0.216
101	0.244

ff =

	0
236	0.066
237	0.532
238	0.463
239	0.188
240	0.094
241	0.138
242	0.185
243	0.296
244	0.073
245	0.543
246	0.162
247	0.105
248	0.056
249	0.262
250	0.105
251	0.401



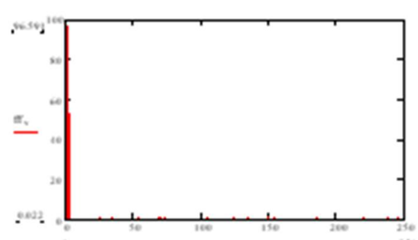


Figure 5 Fragment of the lookup table and corresponding diagrams x0:=0; y0:=0; ξ=20%;

Conclusion:

In the quest to predict and control system behaviors, researchers and engineers rely on mathematical models and numerical simulations. While these simulations provide valuable insights, the initial uncertainty regarding the best choice among alternatives poses a significant challenge.

Engineering optimization endeavors often center around identifying alternatives with optimal properties, leveraging mathematical approaches and optimal control methods. However, the abundance of optimization methods and corresponding software necessitates a judicious evaluation of their effectiveness across various conditions.

Our investigation into the fastest ascent method underscores the importance of selecting appropriate starting coordinates. The method's efficacy in approaching the extremum of the Rosenbrock function is evident, albeit varying with the choice of initial points. Furthermore, the incorporation of error considerations underscores the need for robust methodologies capable of navigating uncertainties.

In conclusion, optimization theory continues to be a driving force behind technological innovation, empowering engineers and researchers to tackle complex challenges and unlock new frontiers in scientific inquiry and technical progress.

References

1. Sergiy Butenko, Panos M. Pardalos Numerical Methods and Optimization An Introduction Edition1st Edition,, First Published, 2014
2. Jamil, Momin, and Xin-She Yang. "A literature survey of benchmark functions for global optimization problems." International Journal of Mathematical Modelling and Numerical Optimization 4.2 (2013)
3. George Lindfield, John Penny Numerical Methods: Using MATLAB· Elsevier Science, 2018
4. Nona Otkhozoria, Vano Otkhozoria, & Shorena Khorava. (2022). Search for an extremum using the steepest descent method under the conditions of experimental errors. World Science, (2(74). https://doi.org/10.31435/rsglobal_ws/28022022/7785
5. Z Azmaiparashvili, N Otkhozoria (2021) “Mathematical Model for Studying the Accuracy Characteristics of Devices for Measuring the Resonant Frequency of Oscillatory Systems New Approaches in Engineering Research” 121-131 https://doi.org/10.9734/bpi/naer/v5/10242D
6. Otkhozoria, N., Narchemashvili, M. and Otkhozoria, V. (2023) “The effectiveness of the Hooke-Jeeves method in the experiment under error conditions is assessed”, Georgian Scientists, 5(4), pp. 285–288. doi: 10.52340/gs.2023.05.04.25.
7. Zaal Azmaiparashvili, Nona Otkhozoria, Alexander Maltsev. Analytic-Imitation Model for Determination of the Natural Frequency of Oscillatory Systems and Its Research. Engineering and Technology. Vol. 4, No. 6, 2017, pp. 82-87. http://www.aascit.org/journal/archive2?journalId=896&paperId=5986
8. Menabde, T., Otkhozoria, N., & Otkhozoria, V. (2024). Use of the theory of measurement uncertainty in procedures for data processing and results obtained by checking-calibration gas flow

meters. International Science Journal of Engineering & Agriculture, 3(2), 40–46. https://doi.org/10.46299/j.isjea.20240302.03

საინჟინრო პრაქტიკული ამოცანების გადაწყვეტისათვის მათემატიკური ოპტიმიზაციის მეთოდების შერჩევა

ნონა ოთხოზორია, ნინო წიკლაური, ვანო ოთხოზორია,
საქართველოს ტექნიკური უნივერსიტეტი

რეზიუმე
ოპტიმიზაციის თეორია გადამწყვეტ როლს თამაშობს თანამედროვე სამეცნიერო და ტექნიკურ მიმართულებებში, რომელიც მოიცავს სხვადასხვა საინჟინრო დისციპლინებს. ოპტიმიზაციის უპირველესი მიზანია მრავალ პოტენციურ შედეგს შორის ყველაზე ოპტიმალური გადაწყვეტის იდენტიფიცირება, სხვადასხვა სტრატეგიების გამოყენებით დაწყებული ანალიტიკური მეთოდოლოგიებიდან რიცხვითი სიმულაციებით დამთავრებული. ეს ნაშრომი იკვლევს უსწრაფესი ასვლის მეთოდის ეფექტურობას როზენბროკის ფუნქციის მაგალითზე, ხაზს უსვამს შესაბამისი საწყისი კოორდინატების არჩევის მნიშვნელობას. გარდა ამისა, ნაშრომში გამოკვლეულია შემთხვევითი ცვლადების მეშვეობით შემოტანილი შეცდომების გავლენა, რაც ხაზს უსვამს მტკიცე მეთოდოლოგიების საჭიროებას, რომელსაც შეუძლია განუსაზღვრელობების შემცირება. ყოვლისმომცველი ანალიზისა და ექსპერიმენტების საშუალებით, ეს კვლევა ხელს უწყობს ოპტიმიზაციის მეთოდოლოგიების ირგვლივ მიმდინარე დისკურსს, ნათელს ჰფენს მათ ეფექტურობასა და გამოყენებადობას მრავალფეროვან საინჟინრო კონტექსტში.

საკვანძო სიტყვები: ოპტიმიზაციის თეორია, საინჟინრო პრაქტიკა, ოპტიმალური მართვის ტექნიკა, უსწრაფესი ასვლის მეთოდი, როზენბროკის ფუნქცია